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Extraction of the proton parton density functions using a NLO-QCD fit of the combined H1 and ZEUS inclusive DIS cross sections

H1 and ZEUS Collaborations

Abstract
The combined HERA-I data set, of neutral and charged current inclusive cross-sections for e^+p and e^-p scattering, is used as the sole input for a next-to-leading order (NLO) QCD parton distribution function (PDF) fit. The consistent treatment of systematic uncertainties in the joint data set ensures that experimental uncertainties on the PDFs can be calculated without need for an increased χ^2 tolerance. This results in PDFs with greatly reduced experimental uncertainties compared to the separate analyses of the ZEUS and H1 experiments. Model uncertainties, including those arising from parametrization dependence, are also considered. The resulting HERAPDFs (called HERAPDF0.1) have impressive precision compared to the present global fits.

1 Introduction

The kinematics of lepton hadron scattering is described in terms of the variables Q^2, the invariant mass of the exchanged vector boson, Bjorken x, the fraction of the momentum of the incoming nucleon taken by the struck quark (in the quark-parton model), and y which measures the energy transfer between the lepton and hadron systems. The differential cross-section for the neutral current (NC) process is given in terms of the structure functions by

$$\frac{d^2\sigma(e^{\pm}p)}{dxdQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[Y_+\, F_2(x,Q^2) - y^2\, F_L(x,Q^2) \mp Y_-\, xF_3(x,Q^2)\right],$$

where $Y_\pm = 1 \pm (1-y)^2$. The structure functions F_2 and xF_3 are directly related to quark distributions, and their Q^2 dependence, or scaling violation, is predicted by perturbative QCD. For low x, $x \leq 10^{-2}$, F_2 is sea quark dominated, but its Q^2 evolution is controlled by the gluon contribution, such that HERA data provide crucial information on low-x sea-quark and gluon distributions. At high Q^2, the structure function xF_3 becomes increasingly important, and gives information on valence quark distributions. The charged current (CC) interactions also enable us to separate the flavour of the valence distributions at high-x, since their (LO) cross-sections are given by,

$$\frac{d^2\sigma(e^+p)}{dxdQ^2} = \frac{G_F^2 M_W^4}{(Q^2+M_W^2)^2 2\pi x} x\left[(\bar{u}+\bar{c}) + (1-y)^2(d+s)\right],$$

$$\frac{d^2\sigma(e^-p)}{dxdQ^2} = \frac{G_F^2 M_W^4}{(Q^2+M_W^2)^2 2\pi x} x\left[(u+c) + (1-y)^2(\bar{d}+\bar{s})\right].$$

Parton Density Function (PDF) determinations are usually obtained in global NLO QCD fits [1–3], which use fixed target DIS data as well as HERA data. In such analyses, the high statistics HERA NC e^+p data have determined the low-x sea and gluon distributions, whereas the fixed target data have determined the valence distributions. Now that high-Q^2 HERA data on NC and CC e^+p and e^-p inclusive double differential cross-sections are available, PDF fits can be made to HERA data alone, since the HERA high Q^2 cross-section data can be used to determine the valence distributions. This has the advantage that it eliminates the need for heavy target corrections, which must be applied to the ν-Fe and μD fixed target data. Furthermore there is no need to assume isospin symmetry, i.e. that d in the proton is the same as u in the neutron, since the d distribution can be obtained directly from CC e^+p data.

The H1 and ZEUS collaborations have both used their data to make PDF fits [4], [5]. Both of these data sets have very small statistical uncertainties, so that the contribution of systematic uncertainties becomes dominant and consideration of point to point correlations between systematic uncertainties is essential. The ZEUS analysis takes account of correlated experimental systematic errors by the Offset Method, whereas H1 uses the Hessian method [6]. Whereas the resulting ZEUS and H1 PDFs are compatible, the gluon PDFs do have rather different shapes, see Fig 6, and the uncertainty bands spanned by these analyses are comparable to those of the global fits.

It is possible to improve on this situation since ZEUS and H1 are measuring the same physics in the same kinematic region. These data have been combined using a 'theory-free' Hessian fit in which the only assumption is that there is a true value of the cross-section, for each process, at each x, Q^2 point [7]. Thus each experiment has been calibrated to the other. This works well because the sources of systematic uncertainty in each experiment are rather different, such that all the sytematic uncertainties are re-evealuated. The resulting systematic uncertainties on each of the combined data points are significantly smaller than the statistical errors. In the present paper this combined data set is used as the input to an NLO QCD PDF fit. The consistency of the input data set and its small systematic uncertainties enables us to calculate the experimental uncertainties on the PDFs using the χ^2 tolerance, $\Delta\chi^2 = 1$. This represents a further advantage compared to the global fit analyses where increased tolerances of $\Delta\chi^2 = 50 - 100$ are used to account for data inconsistencies.

For the present HERAPDF0.1 fit, the role of correlated systematic uncertainties is no longer crucial since these uncertainties are relatively small. This ensures that similar results are obtained using either Offset or Hessian methods, or by simply combining statistical and systematic uncertainties in quadrature. The χ^2 per degree of freedom for a Hessian fit is $553/562$ and for a quadrature fit it is $428/562$. For our central fit we have chosen to combine the 43 systematic uncertainties which result from the separate ZEUS and H1 data sets in quadrature, and to Offset the 4 sources of uncertainty which result from the combination procedure. This results in the most conservative uncertainty estimates on the resulting PDFs. The χ^2 per degree of freedom for this fit is $477/562$.

Despite our conservative procedure the experimental uncertainties on the resulting PDFs are impressively small and a thorough consideration of further uncertainties due to model assumptions is necessary. In section 2 we describe the NLO QCD analysis and model assumptions. In section 3 we give results and in section 4 we give a summary.

2 Analysis

The QCD predictions for the structure functions are obtained by solving the DGLAP evolution equations [8] at NLO in the $\overline{\mathrm{MS}}$ scheme with the renormalisation and factorization scales chosen to be Q^2 [1]. The DGLAP equations yield the PDFs at all values of Q^2 provided they are input as functions of x at some input scale Q_0^2. This scale has been chosen to be $Q_0^2 = 4\mathrm{GeV}^2$ and variation of this choice is considered as one of the model uncertainties. The resulting PDFs are then convoluted with NLO coefficient functions to give the structure functions which enter into the expressions for the cross-sections. The choice of the heavy quark masses is, $m_c = 1.4, m_b = 4.75$GeV, and variation of these choices is included in the model uncertainties. For this preliminary analysis, the heavy quark coefficient functions have been caluclated in the zero-mass variable flavour number scheme. The strong coupling constant was fixed to $\alpha_s(M_Z^2) = 0.1176$ [9], and variations in this value of ± 0.002 have also been considered.

The fit is made at leading twist. The HERA data have a minimum invariant mass of the hadronic system, W^2, of $W^2_{min} = 300$ GeV^2 and a maximum x, $x_{max} = 0.65$, such that they are in a kinematic region where there is no sensitivity to target mass and large-x higher twist contributions. However a minimum Q^2 cut is imposed to remain in the kinematic region where perturbative QCD should be applicable. This has been chosen to be $Q^2_{min} = 3.5$ GeV^2. Variation of this cut is included as one of the model uncertainties.

A further model uncertainty is the choice of the initial parametrization at Q_0^2. Three types of parametrization have been considered. For each of these choices the PDFs are parametrized by the generic form

$$xf(x) = Ax^B(1-x)^C(1 + Dx + Ex^2 + Fx^3), \tag{1}$$

and the number of parameters is chosen by 'saturation of the χ^2', such that parameters D, E, F are only varied if this brings significant improvement to the χ^2. Otherwise they are set to zero.

The first parametrization considered follows that used by the ZEUS collaboration. The PDFs for u valence, $xu_v(x)$, d valence, $xd_v(x)$, total sea, $xS(x)$, the gluon, $xg(x)$, and the difference between the d and u contributions to the sea, $x\Delta(x) = x(\bar{d} - \bar{u})$, are parametrized.

$$xu_v(x) = A_{uv}x^{B_{uv}}(1-x)^{C_{uv}}(1 + D_{uv}x + E_{uv}x^2)$$

$$xd_v(x) = A_{dv}x^{B_{dv}}(1-x)^{C_{dv}}$$

$$xS(x) = A_S x^{B_S}(1-x)^{C_S}$$

$$xg(x) = A_g x^{B_g}(1-x)^{C_g}(1 + D_g x)$$

$$x\Delta(x) = A_\Delta x^{B_\Delta}(1-x)^{C_\Delta}$$

[1] The programme QCDNUM [10] has been used and checked against the programme QCDfit [11].

The total sea $xS = 2x(\bar{u}+\bar{d}+\bar{s}+\bar{c}+\bar{b})$, where $\bar{q} = q_{sea}$ for each flavour, $u = u_v + u_{sea}$, $d = d_v + d_{sea}$ and $q = q_{sea}$ for all other flavours. There is no information on the shape of the $x\Delta$ distribution in a fit to HERA data alone and so this distribution has its parameters fixed, such that its shape is consistent with Drell-Yan data and its normalisation is consistent with the size of the Gottfried sum-rule violation. A suppression of the strange sea with respect to the non-strange sea of a factor of 2 at Q_0^2, is imposed consistent with neutrino induced dimuon data from NuTeV. The normalisation parameters, A_{uv}, A_{dv}, A_g, are constrained to impose the number sum-rules and momentum sum-rule. The B parameters, B_{uv} and B_{dv} are set equal, since there is no information to constrain any difference. Finally this ZEUS-style parametrization has 11 free parameters.

The second parametrization considered follows that of the H1 Collaboration The choice of quark PDFs which are parametrized is different. The quarks are considered as u-type and d-type, $xU = x(u_v + u_{sea} + c)$, $xD = x(d_v + d_{sea} + s)$, $x\bar{U} = x(\bar{u} + \bar{c})$ and $x\bar{D} = x(\bar{d} + \bar{s})$, assuming $q_{sea} = \bar{q}$, as usual. These four (anti-)quark distributions are parametrized separately.

$$xU(x) = A_U x^{B_U} (1-x)^{C_U} (1 + D_U x + E_U x^2 + F_U x^3)$$

$$xD(x) = A_D x^{B_D} (1-x)^{C_D} (1 + D_D x)$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}}$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g}$$

Since the valence distributions must vanish as $x \to 0$, the parameters, A and B are set equal for xU and $x\bar{U}$; $A_U = A_{\bar{U}}$, $B_U = B_{\bar{U}}$; and for xD and $x\bar{D}$; $A_D = A_{\bar{D}}$, $B_D = B_{\bar{D}}$. Since there is no information on the flavour structure of the sea it is also necessary to set $B_{\bar{U}} = B_{\bar{D}}$, such that there is a single B parameter for all four quark distributions. The normalisation, A_g, of the gluon is determined from the momentum sum-rule and the parameters D_U and D_D are determined by the number sum-rules. Assuming that the strange and charm quark distributions can be expressed as x independent fractions, $f_s = 0.33$ and $f_c = 0.15$, of the d and u type sea respectively, gives the further constraint $A_{\bar{U}} = A_{\bar{D}}(1-f_s)/(1-f_c)$, which ensures that $\bar{u} = \bar{d}$ at low x. Finally this H1-style parametrization has 10 free parameters.

The third parametrization we have considered combines the best features of the previous. It has less model dependence than the ZEUS-Style parametrization in that it has fewer asumptions concerning $x\Delta$, and it has less model dependence than the H1-style parametrization in that it does not assume equality of all B parameters. Furthermore, although all types of parametrization give acceptable χ^2 values, this parametrization has the best χ^2 and it gives the most conservative experimental errors. This is the parametrization which we chose for our central fit. The PDFs which are parametrized are xu_v, xd_v, xg and $x\bar{U}$, $x\bar{D}$.

$$xu_v(x) = A_{uv} x^{B_{uv}} (1-x)^{C_{uv}} (1 + D_{uv} x + E_{uv} x^2)$$

$$xd_v(x) = A_{dv} x^{B_{dv}} (1-x)^{C_{dv}}$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}}$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g}$$

The normalisation parameters, A_{uv}, A_{dv}, A_g, are constrained to impose the number sum-rules and momentum sum-rule. The B parameters, B_{uv} and B_{dv} are set equal, $B_{uv} = B_{dv}$ and the B parameters $B_{\bar{U}}$ and $B_{\bar{D}}$ are also set equal, $B_{\bar{U}} = B_{\bar{D}}$, such that there is a single B parameter for the valence and another different single B parameter for the sea distributions. Assuming that the strange and charm quark distributions can be expressed as x independent fractions, $f_s = 0.33$ and $f_c = 0.15$, of the d and u type

Model variation	Standard value	Lower Limit	Upper limit
m_c	1.4	1.35	1.5
m_b	4.7	4.3	5.0
Q^2_{min}	3.5	2.5	5.0
Q^2_0	4.0	2.0	6.0
f_s	0.33	0.25	0.40
f_c	0.10	0.08	0.12

Table 1: Standard values of input parameters and cuts, and the variations consiered to evaluate model uncertainty

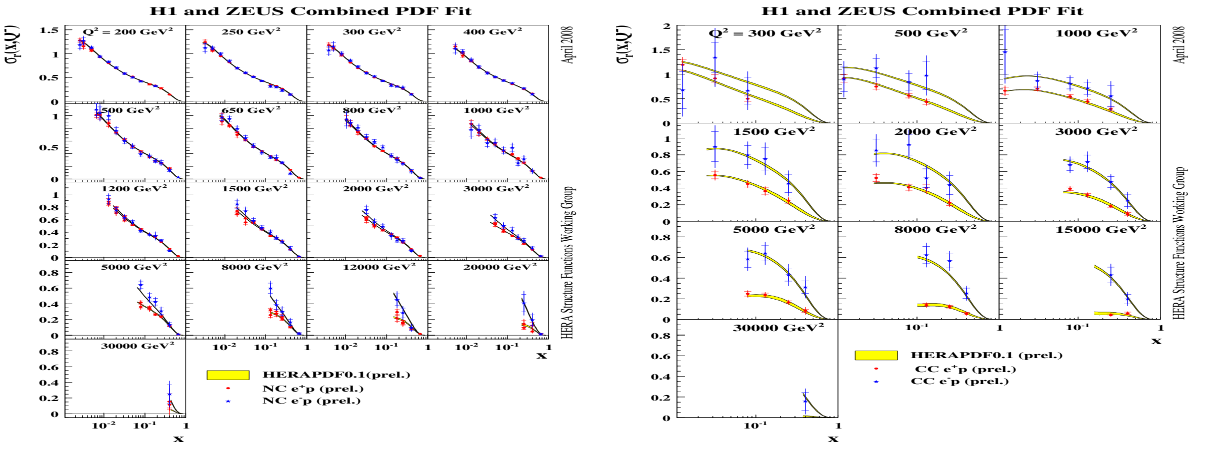

Fig. 1: HERA combined NC (left) and CC (right) data. The predictions of the HERAPDF0.1 fit are superimposed. The uncertainty bands illustrated derive from both experimental and model sources

sea, gives the further constraint $A_{\bar{U}} = A_{\bar{D}}(1 - f_s)/(1 - f_c)$. The value of $f_s = 0.33$ has been chosen to be consistent with determinations of this fraction using neutrino induced di-muon production. This value has been varied to evaluate model uncertainties. The charm fraction has been set to be consistent with dynamic generation of charm from the start point of $Q^2 = m_c^2$, in a zero-mass-variable-flavour-number scheme. A small variation of the value of f_c is included in the model uncertainties. Finally this parametrization has 11 free parameters.

We present results for the HERA PDFs based on the third type of parametrization, including six sources of model uncertainty as specified in Table 1. We also compare to results obtained by varying $\alpha_s(M_Z^2)$ and by varying the choice of parametrization to those of the ZEUS and the H1 styles of parametrization.

3 Results

In Fig. 1 we show the HERAPDF0.1 superimposed on the combined data set for NC data and CC data. In Fig. 2 we show the NC data at low Q^2, and we show the NC reduced cross-section vs. Q^2 for a few representative x bins. The predictions of the HERAPDF0.1 fit are superimposed, together with the predictions of the ZEUS-JETS and H1PDF2000 PDFs. In Fig. 3 we show the NC reduced cross-section vs. Q^2 for the HERA combined data and for fixed target data across the whole of the measured kinematic plane. The predictions of the HERAPDF0.1 fit are superimposed.

Fig. 4 shows the HERAPDF0.1 PDFs, xu_v, xd_v, xS, xg, as a function of x at the starting scale $Q^2 = 4$ GeV^2 and at $Q^2 = 10$ GeV^2. Fig. 5 shows the same PDFs at the scales $Q^2 = 100, 10000$ GeV^2. Fractional uncertainty bands are shown beneath each PDF. The experimental and model uncertainties are

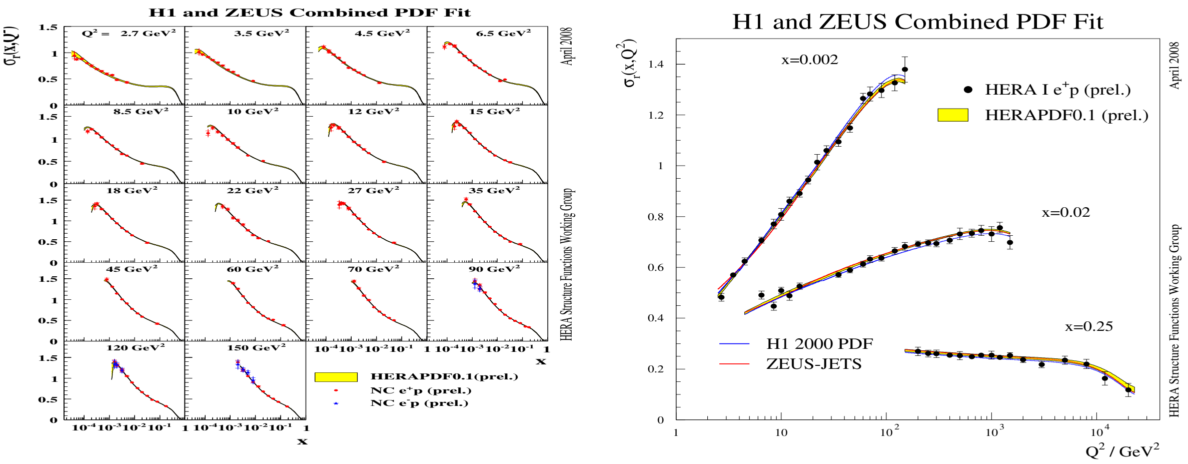

Fig. 2: Left: HERA combined NC data at low Q^2. Right: the NC reduced cross-section vs Q^2 for three x-bins. The predictions of the HERAPDF0.1 fit are superimposed, together with the predictions of the ZEUS-JETS and H1PDF2000 PDFs

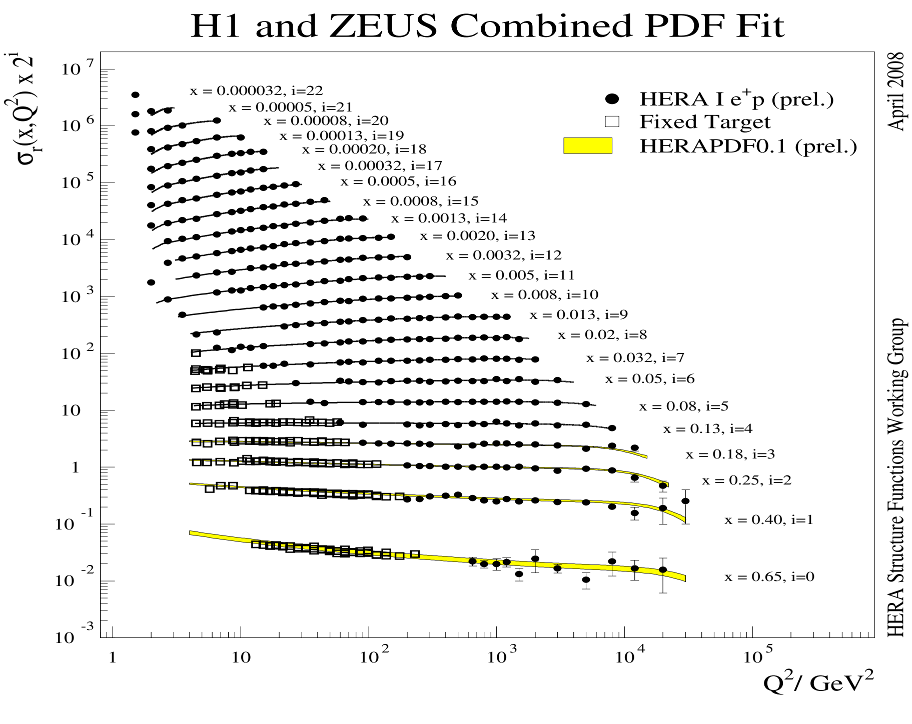

Fig. 3: HERA combined NC data and fixed target data vs Q^2. The predictions of the HERAPDF0.1 fit are superimposed.

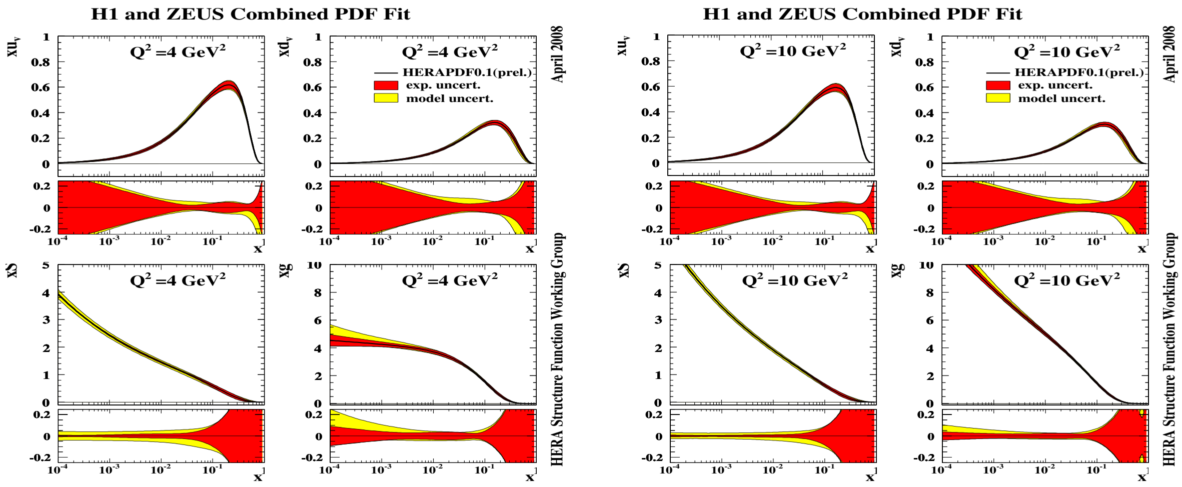

Fig. 4: HERAPDFs, xu_v, xd_v, xS, xg, at (left) $Q^2 = 4$ GeV^2 and (right) $Q^2 = 10$ GeV^2. Fractional uncertainty bands are shown beneath each PDF. The experimental and model uncertainties are shown separately as the red and yellow bands respectively

shown separately. As the PDFs evolve with Q^2 the total uncertainty becomes impressively small.

The total uncertainty of the PDFs obtained from the HERA combined data set is much reduced compared to the PDFs extracted from the analyses of the separate H1 and ZEUS data sets, as can be seen from the summary plot Fig. 6, where these new HERAPDF0.1 PDFs are compared to the ZEUS-JETS and H1PDF2000 PDFs.

The break-up of the HERAPDFs into different flavours is illustrated in Fig. 7, where the PDFs $xU, xD, x\bar{U}, x\bar{D}$ and $x\bar{u}, x\bar{d}, x\bar{c}, x\bar{s}$ are shown at $Q^2 = 10$ GeV^2. The model uncertainty on these PDFs from variation of Q^2_{min}, Q^2_0, m_c and m_b is modest. The model uncertainty from variation of f_s and f_c is also modest except for its obvious effect on the charm and strange quark distributions.

It is also interesting to look at the results obtained from using the ZEUS-style and H1 style parametrizations described in Section 2. In Fig. 8 these alternative parametrizations are shown as a blue line superimposed on the HERAPDF0.1 PDFs. These variations in parametrization produce changes in the resulting PDFs which are comparable to the experimental uncertainties in the measured kinematic range. A further variation of parametrization originates from the fact that, if the D parameter for the gluon is allowed to be non-zero, then each type of parametrization yields a double minimum in χ^2 such that the gluon may take a smooth or a 'humpy' shape. Although the lower χ^2 is obtained for the smooth shape, the χ^2 for the 'humpy' shape is still acceptable. The PDFs for the 'humpy' version of our chosen form of parametrization are compared to the standard version in Fig. 9, where they are shown as a blue line superimposed on the HERAPDF0.1 PDFs. This comparison is shown at $Q^2 = 4\text{GeV}^2$, where the difference is the greatest. Nevertheless the resulting PDFs are comparable to those of the standard choice. This explains a long-standing disagreement in the shape of the gluon obtained by the separate ZEUS-JETS and H1PDF200 analyses. The ZEUS data favoured the smooth shape and the H1 data favoured the 'humpy' shape. However the precision of the combined data set results in PDFs for these shapes which are not significantly different in the measured kinematic region.

It is also interesting to compare the PDFs for the standard choice to those obtained with a different input value of $\alpha_s(M_Z^2)$. The uncertainty on the current PDG value of $\alpha_s(M_Z^2)$ is ± 0.002 and thus we vary our central choice by this amount. The results are shown in Fig. 10, where we can see that this variation only affects the gluon PDF, such that the larger(smaller) value of $\alpha_s(M_Z^2)$ results in a harder(softer) gluon as predicted by the DGLAP equations. The change is outside total uncertainty bands of the standard fit. Finally, Figs. 11 and 12 compare the HERAPDF0.1 PDFs to those of the CTEQ and the MRST/MSTW groups respectively. The precision of the HERAPDF0.1 analysis is impressive.

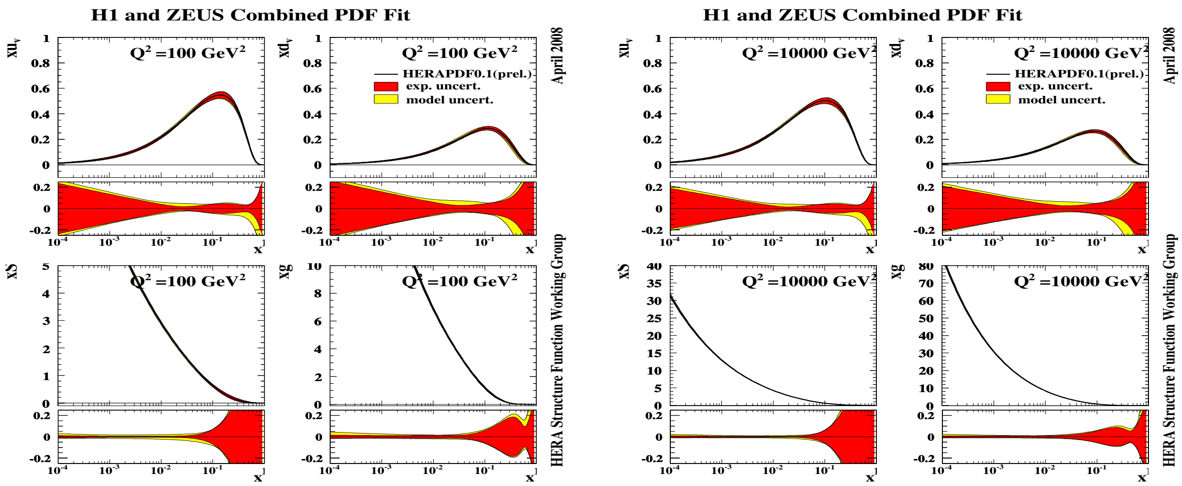

Fig. 5: HERAPDFs, xu_v, xd_v, xS, xg, at (left) $Q^2 = 100$ GeV^2 and (right) $Q^2 = 10000$ GeV^2. Fractional uncertainty bands are shown beneath each PDF. The experimental and model uncertainties are shown separately as the red and yellow bands respectively

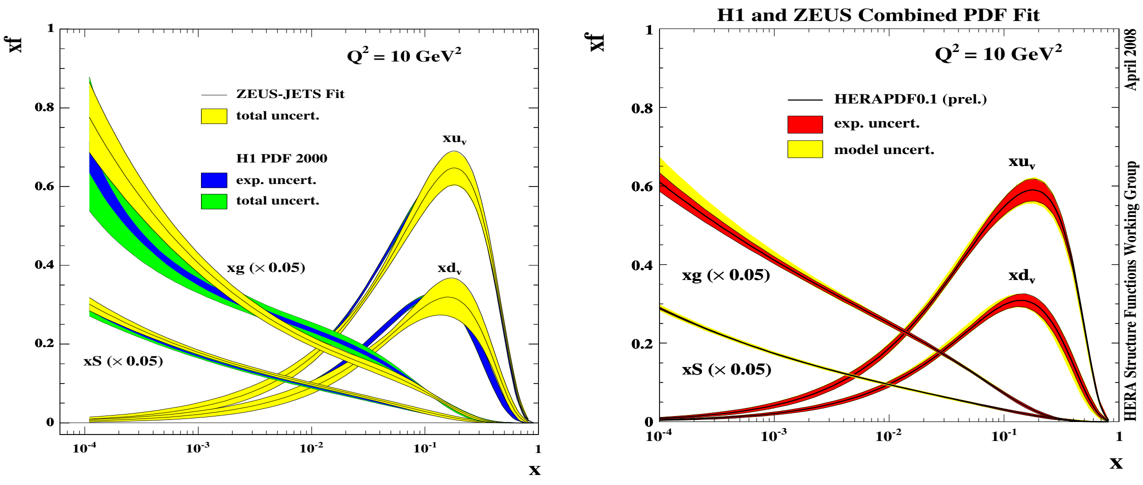

Fig. 6: Left: PDFs from the ZEUS-JETS and H1PDF2000 PDF separate analyses of ZEUS and H1. Right: HERAPDF0.1 PDFs from the analysis of the combined data set

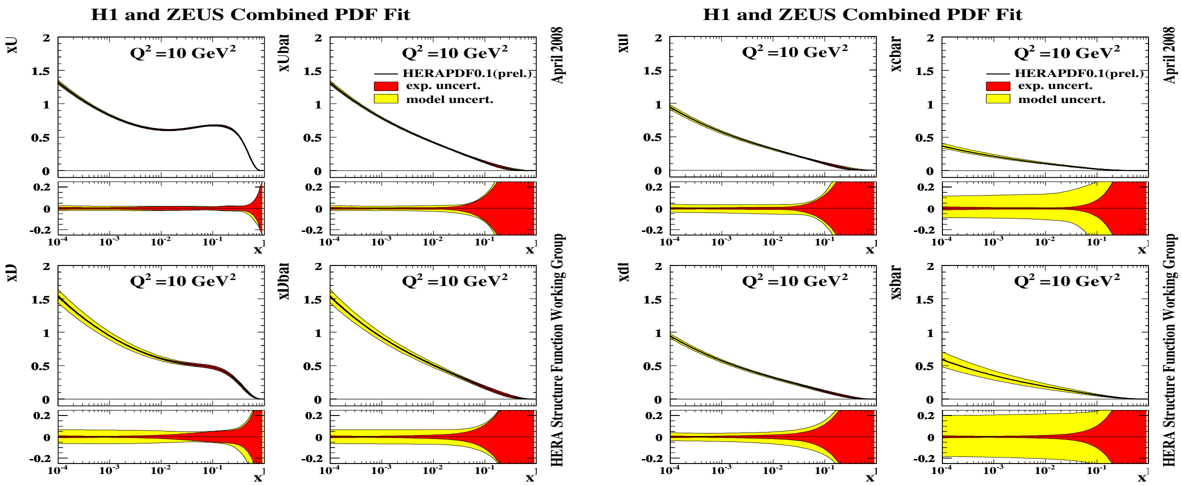

Fig. 7: HERAPDFs at $Q^2 = 10\text{GeV}^2$: (left) $xU, xD, x\bar{U}, x\bar{D}$; (right) $x\bar{u}, x\bar{d}, x\bar{c}, x\bar{s}$. Fractional uncertainty bands are shown beneath each PDF. The experimental and model uncertainties are shown separately as the red and yellow bands respectively

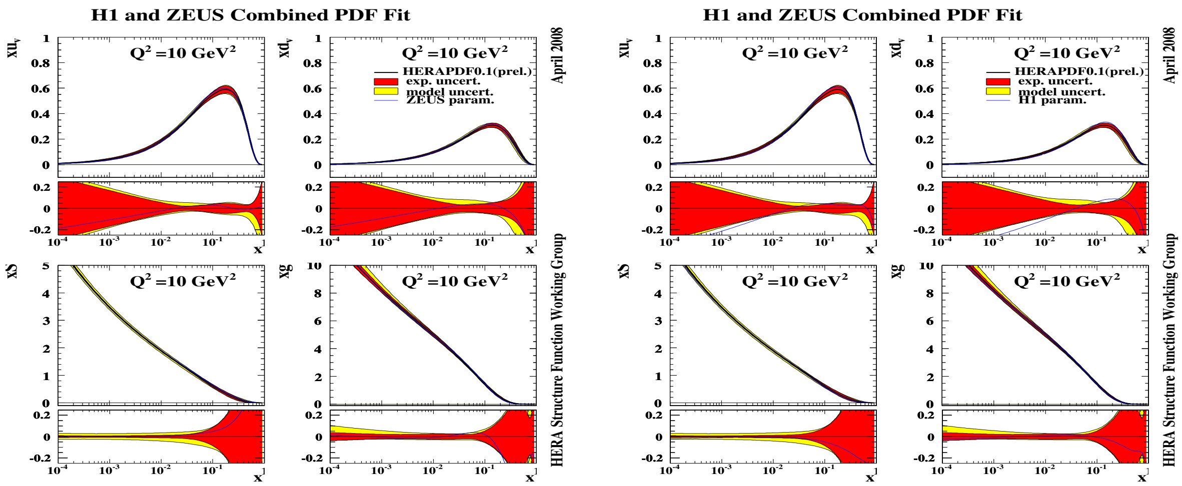

Fig. 8: HERAPDFs at $Q^2 = 10\text{GeV}^2$: with the results for the ZEUS-style parametrization (left) and for the H1-style parametrization (right) superimposed as a blue line.

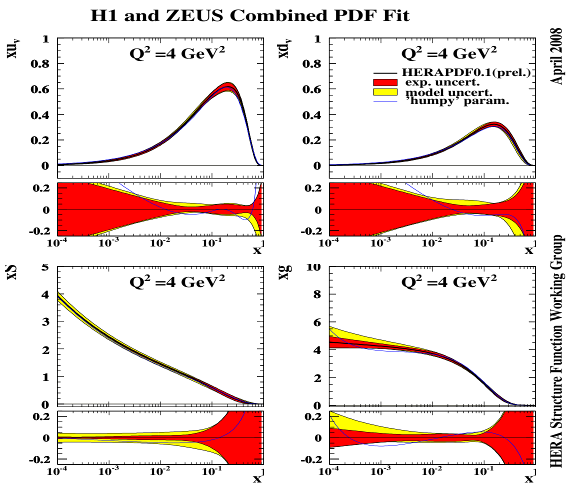

Fig. 9: HERAPDFs at $Q^2 = 4\text{GeV}^2$: with the results for the humpy version superimposed as a blue line.

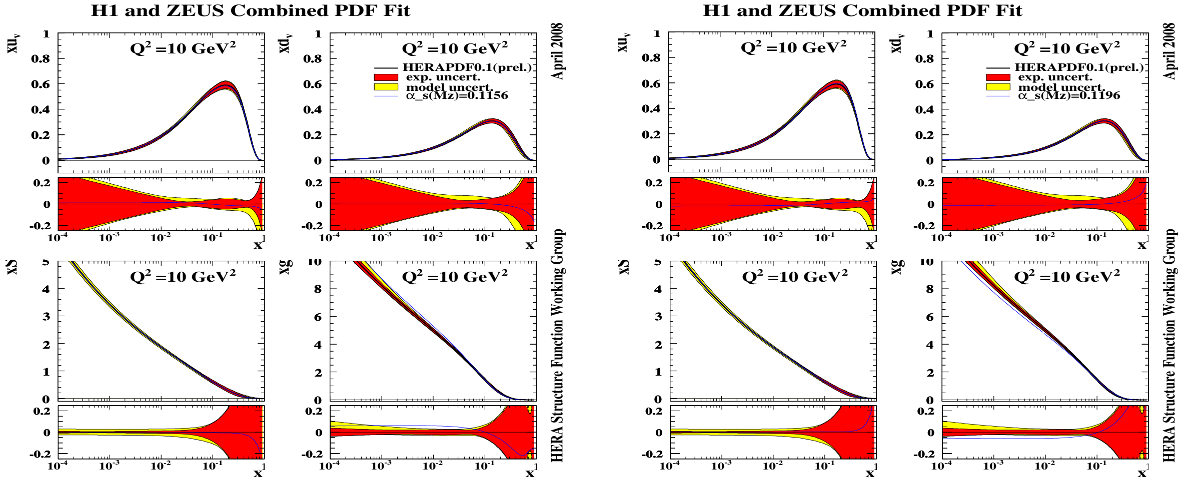

Fig. 10: HERAPDFs at $Q^2 = 10\mathrm{GeV}^2$: with the results for $\alpha_s(M_Z^2) = 0.1156$ (left) and for $\alpha_s(M_Z^2) = 0.1196$ (right) supermiposed as a blue line.

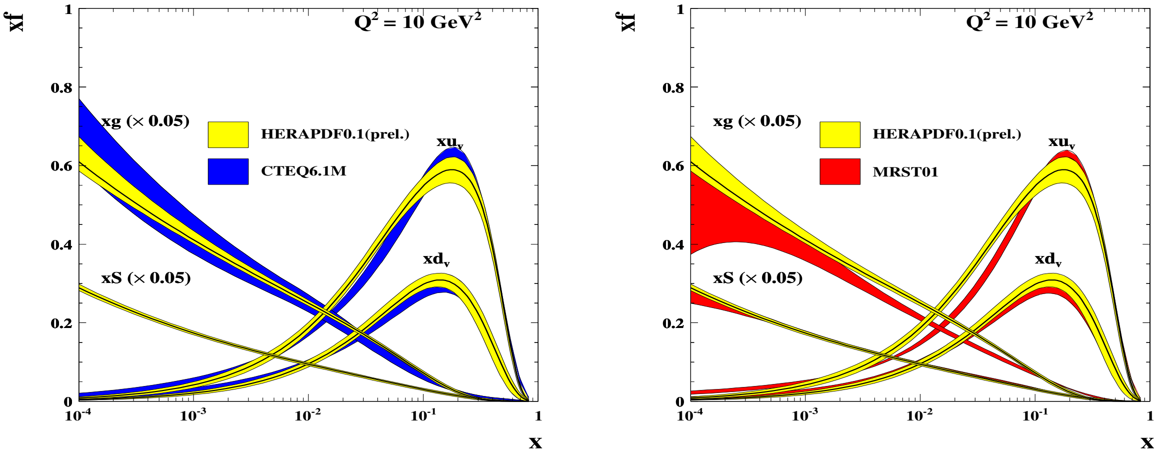

Fig. 11: HERAPDFs at $Q^2 = 10\mathrm{GeV}^2$ compared to the PDFs from CTEQ6.1 and MRST01

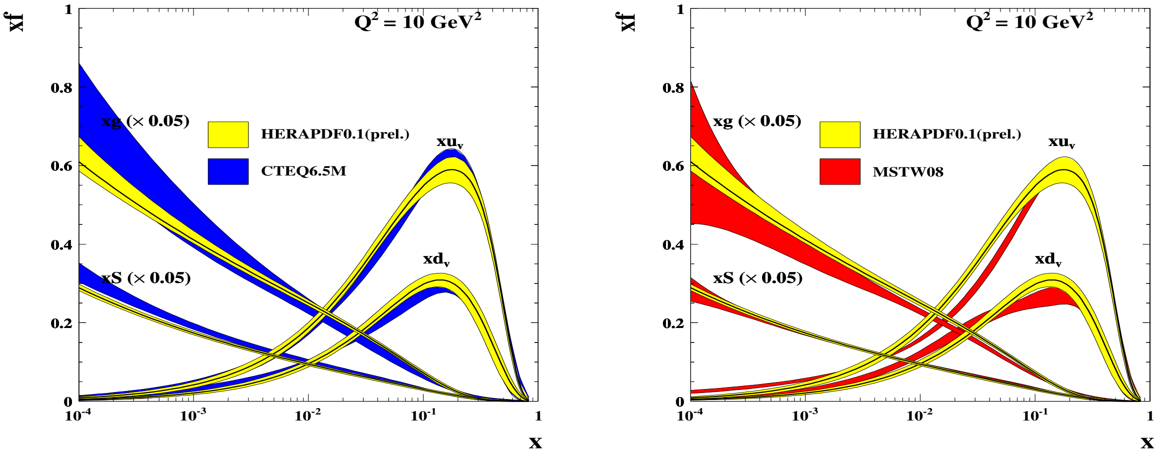

Fig. 12: HERAPDFs at $Q^2 = 10\mathrm{GeV}^2$ compared to the PDFs from CTEQ6.5 and MSTW08(prel.)

4 Summary

The combined HERA-I data set, of neutral and charged current inclusive cross-sections for e^+p and e^-p scattering, provides high precision data, with small systematic uncertainties, across a broad kinematic range such that these data can be used as the sole-input for an NLO QCD PDF fit. The data at low-x, $x < 0.01$, provide information on the Sea and the gluon PDFs, and the high-Q^2 HERA data can be used to determine the valence PDFs. The consistent treatment of systematic uncertainties in the combined data set ensures that experimental uncertainties on the PDFs can be calculated without need for an increased χ^2 tolerance. This results in PDFs with greatly reduced experimental uncertainties compared to the separate analyses of the ZEUS and H1 experiments. Model uncertainties, including those arising from parametrization dependence, have also been carefully considered. The resulting HERAPDFs (called HERAPDF0.1) have impressive precision compared to the global fits. These PDFs will be released on LHAPDF.

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