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Extraction of the Charm Contribution to the Proton Structure, $F_2^{c\bar{c}}$, from $D^{*\pm}$ Measurement in Deep Inelastic Scattering at HERA

H1 Collaboration

Abstract

The charm contribution to the proton structure, $F_2^{c\bar{c}}\left(x, Q^2\right)$, is determined using the inclusive cross sections of $D^{*\pm}(2010)$ meson production in deep-inelastic ep scattering measured by the H1 collaboration at HERA. The data taken during the HERA-II running period correspond to an integrated luminosity of 347 pb^{-1}. The inclusive $D^{*\pm}$ meson cross section has been measured in the range $5 < Q^2 < 1000\,\mathrm{GeV}^2$, $0.02 < y < 0.7$, $p_{t\,D^*} > 1.5$ GeV and $|\eta_{D^*}| < 1.5$. The charm contribution to the proton structure, $F_2^{c\bar{c}}\left(x, Q^2\right)$, is determined for the kinematic region 7 GeV^2 $\leq$ Q^2 $\leq$ 440 GeV^2 and $10^{-4} < x_{bj} < 3 \times 10^{-2}$ by extrapolating the visible $D^{*\pm}$ meson cross section to the full phase space using a next-to-leading order QCD calculation based on DGLAP evolution and a leading order Monte-Carlo program including parton showers based on CCFM evolution.

1 Introduction

Results on $D^{*\pm}$ meson production in deep-inelastic ep scattering (DIS) and on the charm contribution to the proton structure function, $F_2^{c\bar{c}}$, at HERA have been published by the H1 and the ZEUS collaborations [1, 2, 3]. These data have shown clear evidence that the dynamics of charm production in ep scattering is dominated by the photon gluon fusion process. In this framework the process $e^+p \rightarrow e^+D^{*\pm}X$ is sensitive to the gluon density in the proton [4] and allows its universality to be tested.

The early results from the H1 experiment [1] based on an integrated luminosity of 18.6 pb^{-1} collected during the 1996 and 1997 HERA running period were statistically limited. The current analysis uses data from the full HERA-II running period, yielding an almost 20 times larger integrated luminosity of 347 pb^{-1}. Hence, more precise tests of perturbative QCD (*pQCD*) become possible.

2 Models of Open Charm Production

2.1 NLO Calculations of charm production

The description of open heavy flavour production in electron proton collisions is based on perturbative QCD. In leading order (LO), the photon gluon fusion process (*BGF*), i.e. $\gamma g \rightarrow Q\overline{Q}$, is the dominant contribution [1]. This treatment has been extended to next-to-leading order (NLO) for which calculations in several schemes are available [5, 6, 7, 8, 9]. All approaches assume the scale to be hard enough to apply pQCD and to guarantee the validity of the factorisation theorem.

Here, the "massive approach" is adopted, i.e. a fixed order calculation with massive quarks assuming three active flavours in the proton. The momentum densities of the three light quarks and the gluon in the proton are evolved by the DGLAP equation [10]. The heavy quarks are assumed to be produced only at the perturbative level [5] via photon gluon fusion. These calculations are considered reliable in the regime $Q^2 \approx m_Q^2$. However, they are assumed to break down at some scale $Q^2 \gg m_Q^2$ due to missing resummation of large logarithms $\sim \ln(Q^2/m_Q^2)$.

Based on the NLO calculations of order α_s^2 in the coefficient functions[5] programs for different applications were developed in the fixed-flavour-number-scheme (FFNS). The Riemersma program [6] allows the calculation of inclusive quantities of heavy quark production, like $F_2^{c\bar{c}}(x, Q^2)$, while the HVQDIS program [7, 11] allows the calculation of exclusive quantities by providing the four-momenta of the outgoing partons. Both programs are based on the same calculation and use same inputs like PDF's, charm mass and factorisation and renormalization scales. In this analysis the MRST04FF3nlo[25] parton density will be used with a charm mass of m_c =1.43 GeV together with the renormalization and factorisation scales $\mu = \mu_r = \mu_f = \sqrt{Q^2 + 4m_c^2}$ as default.

Visible differential cross sections for the inclusive production of charmed meson are obtained by fragmenting independently the charm quarks into $D^{*\pm}$ mesons in the photon - proton centre of mass frame. The longitudinal fragmentation function has to be non-perturbative in order to avoid double-counting of diagrams which may appear in the NLO calculation of the boson-gluon-fusion process as well as in the evolution of the fragmentation process. For this purpose the Kartvelishvili fragmentation function [12] which is controlled by a single parameter α is chosen in this analysis. In addition, to account for the experimentally observed p_t smearing of hadrons with respect to the quark direction, the $D^{*\pm}$ meson has been given a transverse momentum p_t with respect to the charm quark, according to the function $\exp(-ap_t^2)$. The parameter a is chosen such that an average transverse momentum $\langle p_t \rangle \approx 350$ MeV is obtained as observed in e^+e^- data [13].

2.2 Charm production based on CCFM evolution

The CCFM evolution equation [15] is expected to be more appropriate to describe the parton evolution at small x, where the collinear assumption of a large longitudinal momentum compared to the transverse momentum of the partons might not be valid. In the parton cascade, gluons are emitted in an angular ordered manner to account for coherence effects. Due to this angular ordering, the gluon distribution depends on the maximum allowed angle in addition to the momentum fraction x and the transverse momentum of the propagator gluon. The cross section is then calculated according to the k_t-factorisation theorem by convoluting the unintegrated gluon density with the off-shell photon gluon fusion matrix element with massive quarks for the hard scattering process.

Visible $D^{*\pm}$ meson cross sections as well as the charm contribution to the proton structure function $F_2^{c\overline{c}}$ are calculated with the Monte-Carlo program CASCADE [16]. The parametrization set-A0 [17] is used for the unintegrated gluon distribution of the proton. It has been determined from a fit to inclusive F_2 data published by H1 [18] and ZEUS [19]. The renormalization scale is set to $\mu_r^2 = 4m_c^2 + Q^2$. Time-like parton showers off the charm quark and anti-quark are implemented. The harmonization of partons is performed using the Lund String model as implemented in PYTHIA [20]. Higher charm resonances as determined by ALEPH [21] are included. For the longitudinal fragmentation of the charm quark into the D^* meson the Kartvelishvili parametrization is also used here.

2.3 Fragmentation parameters

In a recent publication the H1 collaboration has extracted the parameters of non-perturbative charm fragmentation functions for $D^{*\pm}$ mesons within different models [14]. It turned out that it is not possible to describe the measured distributions of fragmentation variables with a single non-perturbative fragmentation parameter in any model investigated in the full region of the visible phase space for $D^{*\pm}$ meson production. A significantly harder charm fragmentation is observed in data close to the charm production threshold than predicted by the models based on the parameters extracted from the $D^{*\pm}$ meson production data significantly above threshold. This inconsistency points to a lack of the current understanding of charm fragmentation in the visible phase space accessible in deep-inelastic ep scattering at HERA.

To cope with this observation in the extraction of $F_2^{c\overline{c}}(x, Q^2)$ different non-perturbative fragmentation parameters will be used close to and well above threshold according to the measurement [14] when calculating the expected visible cross section of $D^{*\pm}$ meson production. The sets of parameters for HVQDIS and CASCADE are summarized in table 1. A cut in the invariant mass $\hat{s}$ of the $c\overline{c}$ system is applied to define the two samples such that the mean values of $\hat{s}$ of the two samples agree with those of the measurements.

Model	Kartvilishvili Parameter α	
	small $\hat{s}$ region	large $\hat{s}$ region
HVQDIS	$6.0^{+1.0}_{-0.8}$	3.3 ± 0.4
CASCADE	$8.2^{+1.2}_{-1.1}$	$4.5^{+0.6}_{-0.5}$

Table 1: Fragmentation parameters α of the Kartvilishvili fragmentation for small $\hat{s}$ and large $\hat{s}$ used for inclusive $D^{*\pm}$ meson cross section calculations of the HVQDIS and CASCADE program.

3 Data used for the extraction of $F_2^{c\bar{c}}$

The data have been collected with the H1 detector [22] at HERA during the running periods 2004-2007 when HERA operated with 27.5 GeV positrons or electrons and 920 GeV protons colliding at a centre of mass energy of $\sqrt{s} = 318$ GeV. H1 Preliminary results [23, 24] on the visible D^* inclusive cross sections at medium and high Q^2 are used.

3.1 Differential Cross Sections

Figure 1 shows the inclusive double differential $D^{*\pm}$ cross sections in the visible region as a function of y in bins of Q^2 together with the expectations from the HVQDIS program with the settings defined in Sec.2.1.

The shaded bands represent the results of a variations of the input parameters for the NLO calculation. The charm quark mass has been varied from $m_c = 1.3$ GeV to $m_c = 1.6$ GeV, the renormalization and factorisation scales have been varied simultaneously $0.5\mu < \mu_r = \mu_f < 2\mu$. The cut in $\hat{s}$ to define the two different samples in Tab.1 for the fragmentation treatment described in Sec.2.3 was varied by 20 GeV^2 around the central value of $\hat{s}$ =70 GeV^2. The light yellow shaded band represent the full theory uncertainty, obtained from the variations of the charm mass, renormalization and factorisation scales and the fragmentation parameters. The dark yellow shaded area shows the theory uncertainty alone from the variations in the fragmentation parameters.

Although the predicted visible cross section is smaller than experimentally observed, the agreement with the data in the shapes of the different single differential cross sections is reasonable.

In Fig. 2 the same double differential $D^{*\pm}$ cross sections are compared to the CASCADE expectations with the parameter settings as described in Sec.2.2 and in Sec.2.3. Also here reasonable agreement is observed.

4 Charm Contribution $F_2^{c\bar{c}}$ to the Proton Structure Function

The charm contribution, $F_2^{c\bar{c}}(x, Q^2)$, to the proton structure function is obtained by using the expression for the one photon exchange cross section for charm production

$$\frac{d^2\sigma^c}{dxdQ^2} = \frac{2\pi\alpha_{em}^2}{Q^4x}\left(1 + (1-y)^2\right)\ F_2^{c\bar{c}}(x, Q^2)\ , \tag{1}$$

The y range of the measurements is chosen such that the contribution from the longitudinal structure function F_L^c is negligible everywhere.

The visible inclusive $D^{*\pm}$ cross sections $\sigma_{\rm vis}^{\rm exp}(x, Q^2)$ in bins of x and Q^2 are converted to a bin centre corrected $F_2^{c\bar{c}\ \rm exp}(\langle x\rangle, \langle Q^2\rangle)$ by the relation:

$$F_2^{c\bar{c}\ \rm exp}(\langle x\rangle, \langle Q^2\rangle) = \frac{\sigma_{\rm vis}^{\rm exp}(x, Q^2)}{\sigma_{\rm vis}^{\rm theo}(x, Q^2)} \cdot F_2^{c\bar{c}\ \rm theo}(\langle x\rangle, \langle Q^2\rangle)\ , \tag{2}$$

where $\sigma_{\rm vis}^{\rm theo}$ and $F_2^{c\bar{c}\ \rm theo}$ are the theoretical predictions from the model under consideration. Following the same line as in previous publications [1, 2, 4] the HVQDIS program by Harris and Smith and the program of Riemersma et al. [5] are used to calculate these quantities in NLO. For the CCFM case the CASCADE Monte-Carlo program is used. The contribution of open beauty production to the visible $D^{*\pm}$ meson cross sections is expected to be of the order of 1-2% and is therefore neglected.

The extraction of $F_2^{c\bar{c}}$ according to Eqn.2 is faced with the problem that the measurement covers only about 20% to 60% of the total phase space for charm production, with the lowest acceptance at large x in the low Q^2 region. Thus the acceptance may depend significantly on the underlying model. To be more explicit, two different calculations may yield the same value for the visible cross section $\sigma_{\rm vis}^{\rm theo}(x, Q^2)$ and give an equally good description of the data, but may have different acceptances leading to different predictions of $F_2^{c\bar{c}\ \rm theo}(\langle x\rangle, \langle Q^2\rangle)$ for a given bin in x and Q^2 and thereby yielding different values for $F_2^{c\bar{c}\ \rm exp}$.

To investigate this model dependence $F_2^{c\bar{c}\ \rm exp}$ is determined using the NLO DGLAP calculation HVQDIS and the LO Monte-Carlo program CASCADE based on the CCFM evolution. Both models give a reasonable description of the measured differential cross sections not only in x and Q^2, but also in the transverse momentum $p_T(D^*)$ and in the pseudo-rapidity $\eta(D^*)$ of the D^* meson. From the theoretical point of view HVQDIS has the advantage of using the boson-gluon fusion matrix element in NLO, while CASCADE uses LO only. On the other hand CASCADE includes incoming gluons that are off-mass shell and have a finite transverse momentum, and in addition approximates higher orders by parton showers. At large x however the disregard of quarks in the parton ladder in CASCADE might not be a valid assumption. The ratio of the extrapolation factors from the visible range to the full phase space in CASCADE and HVQDIS is shown in Fig. 3. Especially at large x and low Q^2, where the extrapolation is largest, the two models differ nearly by a factor of 2. But also in the other bins differences of 10-20% are observed.

4.1 Extrapolation uncertainties

To estimate the effect of extrapolation uncertainties from the theory on the measurement of the $F_2^{c\bar{c}}$ the following parameters have been varied for both, the HVQDIS calculation and the CASCADE program

- the charm quark mass m_c between 1.3 and 1.6 GeV,
- the renormalization scale in the range $0.5\mu < \mu_r < 2\mu$, in case of the HVQDIS calculation simultaneously the factorisation scale, i.e. $\mu_r = \mu_f$,
- the fragmentation parameters according to the errors given in Tab.1,

Parameter	Range	HVQDIS [%]	CASCADE [%]
m_c	1.3 - 1.6 GeV	6	2 - 7
μ_r	0.5μ - 2μ	2-7	up to 2
α	Tab.1	1 - 10	up to 6
$\hat{s}$ cut	50-90 GeV^2	0.5 - 2	3
PDF		1 - 13	-

Table 2: Extrapolation uncertainties in F_2^c for HVQDIS and CASCADE.

- the upper cut in $\hat{s}$ for the definition of the close to threshold region between 50 GeV^2 and 90 GeV^2 and
- in case of the HVQDIS program the parton density MRST04FF3nlo was replaced by the CTEQ5F3[27] PDF.

The effect of these variations on the extrapolation is summarized in Tab. 2. Some comments should be made to these uncertainty estimations.

- The PDF's applicable to HVQDIS and CASCADE have been determined for a given charm quark mass m_c. Charm production contributes to the total inclusive DIS cross section by about 20% in the kinematic region where the exact treatment of the charm mass leads to sizable differences in the PDF extraction and $F_2^{c\bar{c}}$ predictions. Any variation of the charm quark mass in the calculation from the value used for the PDF extraction overestimates the uncertainty on $F_2^{c\bar{c}}$ because the constraint to reproduce the inclusive cross section measurement will not be used this way.
- For HVQDIS the scale variation has been done simultaneously for μ_r and μ_f setting $\mu_r = \mu_f$. If the prescription given in Ref.[26] is followed by applying the constraint $0.5 < \mu_r/\mu_f < 2$ significantly larger relative extrapolation uncertainties of up to about 20% on average are expected.
- In case of CASCADE different PDF's are available depending on the choice of the renormalization scale. Due to the proper choice of the PDF when varying μ_r the extrapolation uncertainties obtained for CASCADE are significantly smaller than for the HVQDIS calculation.

5 Results

Figure 4 shows the charm contribution to the structure function $F_2^{c\bar{c}}$ as a function of x for different values of Q^2 as extracted from the inclusive $D^{*\pm}$ cross sections. The extraction is based on the NLO calculation as implemented in HVQDIS. A charm quark mass m_c of 1.43 GeV and the gluon density distribution MRST04FF3nlo has been used. The total experimental error is obtained by adding in quadrature the statistical and experimental systematic errors and the extrapolation uncertainties due to the variation of the fragmentation model parameters given in Tab.2. The NLO predictions for $m_c = 1.43$ GeV using the gluon distribution MRST04FF3nlo and CTEQ5F3 are also shown by the solid lines. The shaded band on the bottom of each figure gives the Extrapolation uncertainties due to the PDF variation (inner shaded area) and due to the variation of m_c, and the scales $\mu_r = \mu_f$ (outer shaded area) when adding up the different contributions in quadrature.

Figure 5 shows the result on the structure function $F_2^{c\bar{c}}$ from the inclusive $D^{*\pm}$ cross section measurements in the framework of CASCADE accordingly. The solid line gives the prediction of this model using the unintegrated gluon distribution A0. The shaded band on the bottom of each figure shows the effect of the extrapolation uncertainties due to the variation of m_c and μ_r added in in quadrature.

A comparison of Figs. 4 and 5 shows differences in the extracted $F_2^{c\bar{c}}$ for the two different models, especially at large x where the extrapolation is large.

6 Conclusions

The contribution to the proton structure $F_2^{c\bar{c}}$ has been extracted from the measured $D^{*\pm}$ meson production cross section for the kinematic region 7 GeV^2 $\leq Q^2 \leq 440$ GeV^2 and $10^{-4} < x_{bj} < 3 \times 10^{-2}$. The extrapolation from the visible $D^{*\pm}$ meson phase space to the full charm production phase space has been performed in the NLO DGLAP scheme as implemented in HVQDIS and with the LO Monte Carlo program CASCADE based on the CCFM evolution equations. The effect on the extrapolation factors by varying the model parameters has been investigated. The differences in the extracted F_2^c due to parameter uncertainties in the theoretical calculations and especially due to the different schemes of these calculations turned out to be sizable and in general larger than the total experimental uncertainties.

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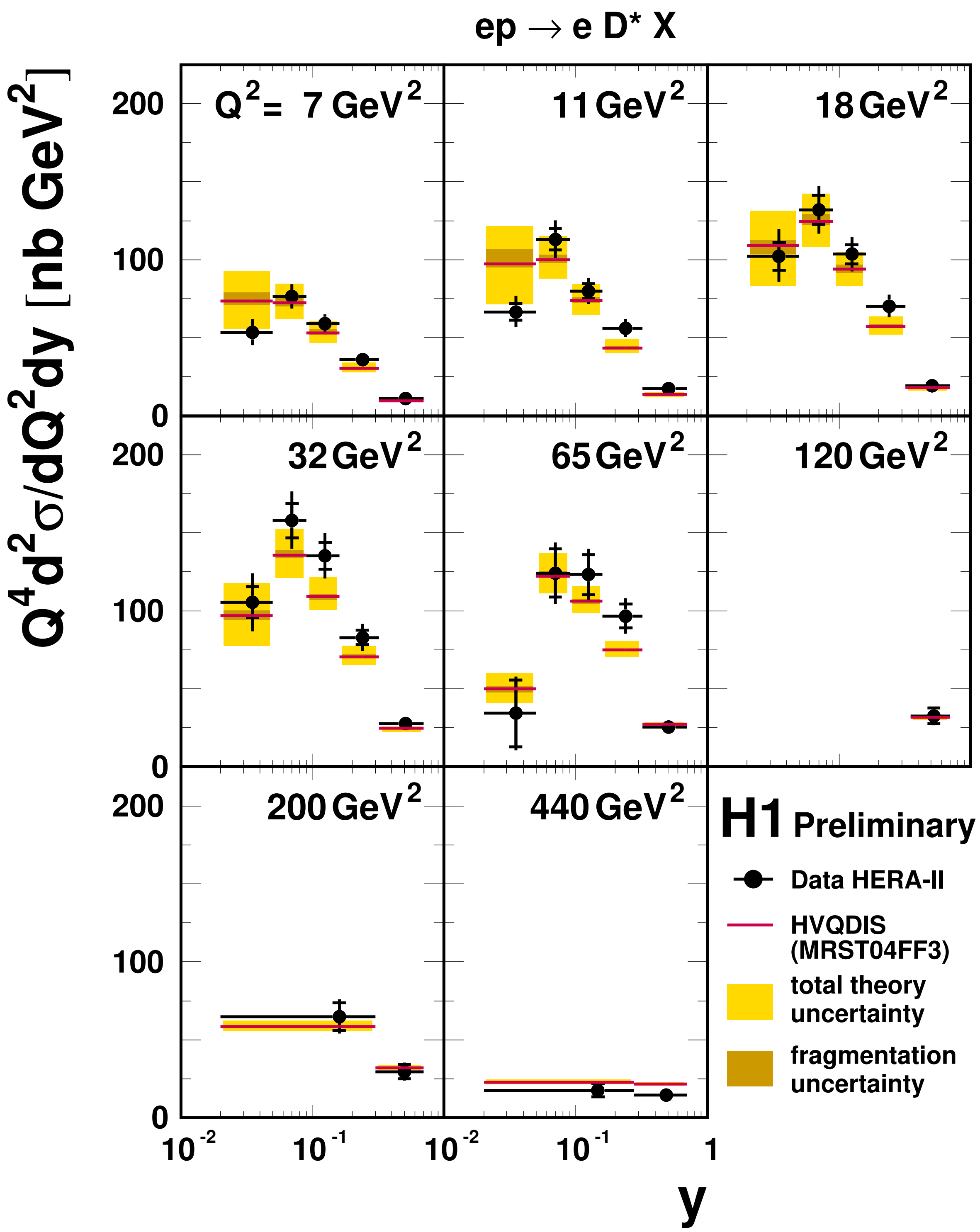

Figure 1: Double differential inclusive cross section $\mathrm{d}^2\sigma/\mathrm{d}Q^2\mathrm{d}y$, The inner and outer thin error bars correspond to the statistical and the total error, respectively. The expectation of the NLO DGLAP calculation using HVQDIS with MRST04FF3nlo parton densities is indicated by the red solid line. The light yellow shaded band represent the full theory uncertainty, obtained from the variations of the charm mass, renormalization and factorisation scales and the fragmentation model. The dark yellow shaded area shows the theory uncertainty alone from the variations in the fragmentation model.

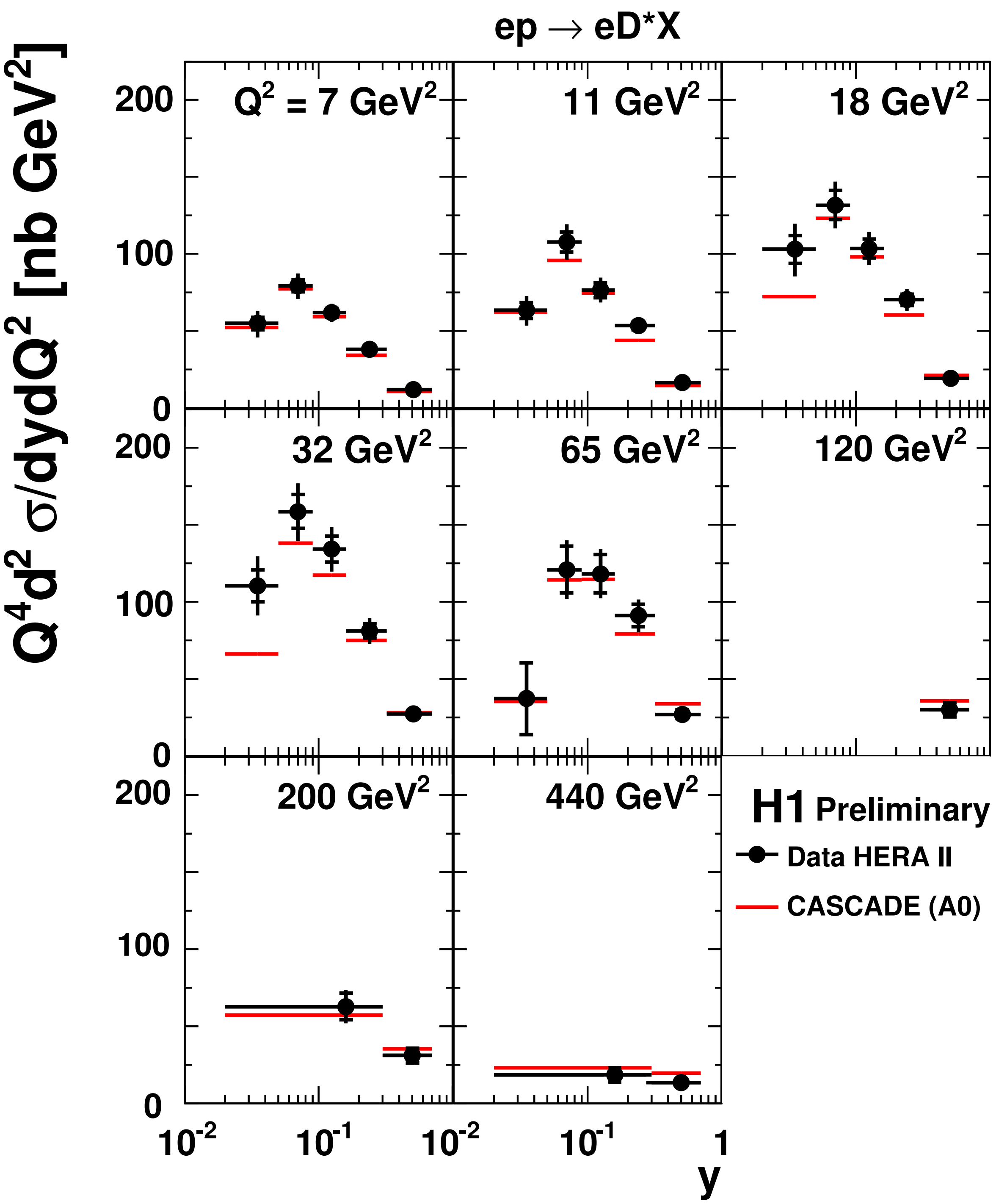

Figure 2: The same double differential $D^{*\pm}$ cross section $\mathrm{d}^2\sigma/\mathrm{d}Q^2\mathrm{d}y$ as in figure 1, but compared to the CASCADE expectation.

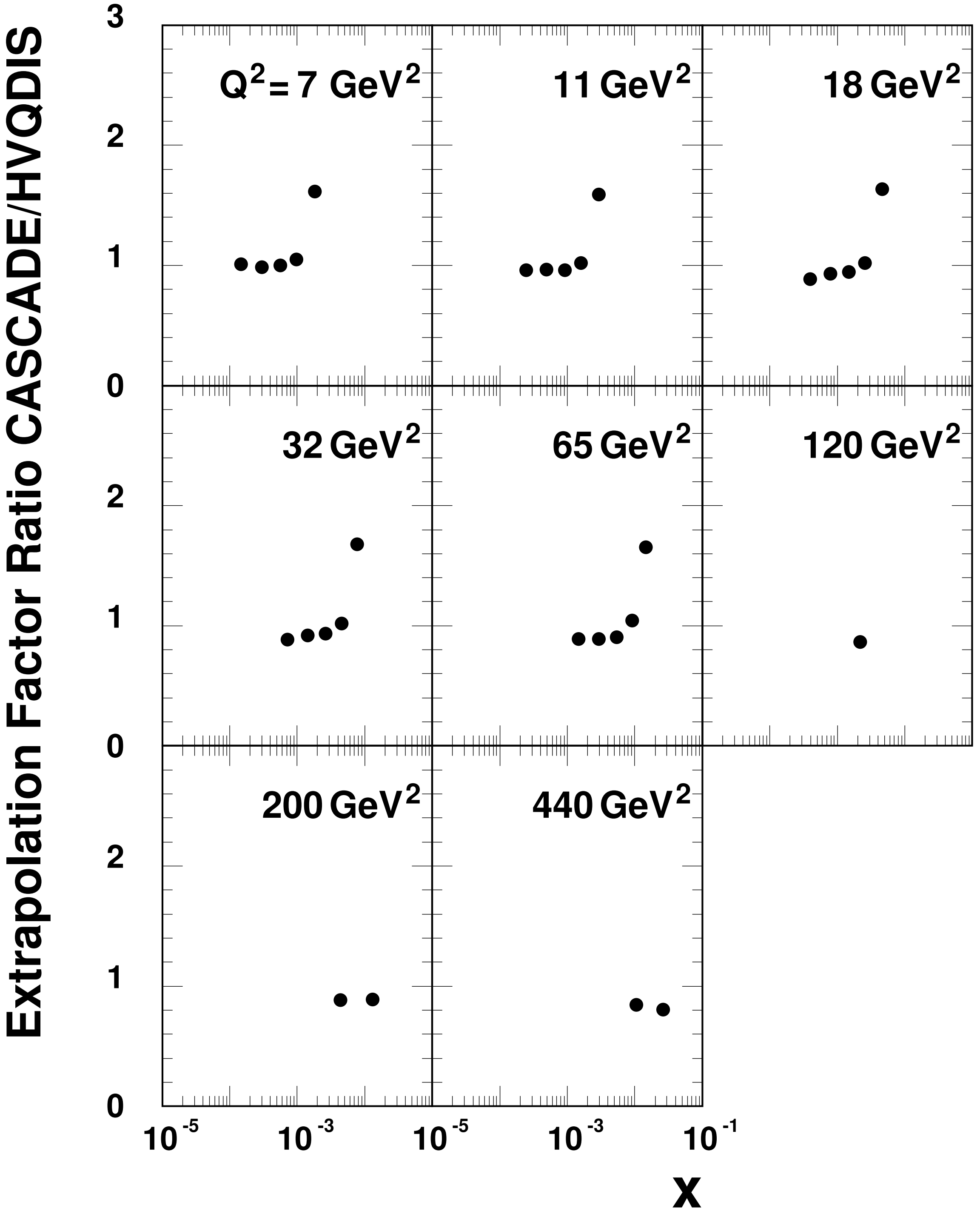

Figure 3: Ratio of the extrapolation factor from the visible range to the full phase space between CASCADE and HVQDIS in bins of Q^2 as a function of x

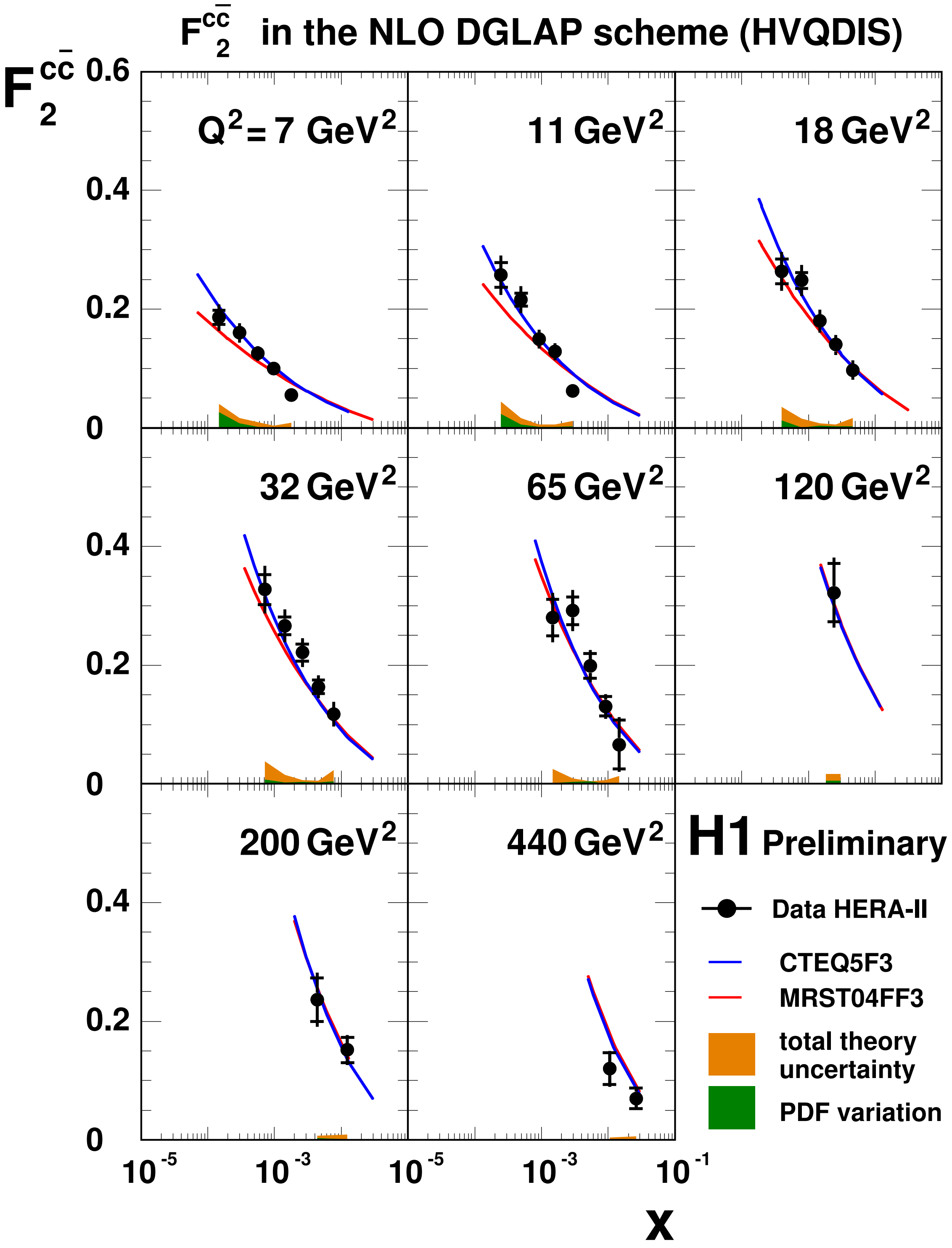

Figure 4: The charm contribution to the proton structure function, $F_2^{c\bar{c}}$, as derived from the inclusive $D^{*\pm}$ meson analysis in the framework of NLO DGLAP using MRST04FF3nlo parton densities at for $m_c = 1.43$ GeV and renormalization and factorisation scales $\mu_r = \mu_f = \sqrt{Q^2 + 4m_c^2}$. The error bars refer to the statistical (inner) and the total (outer) error, respectively. The solid lines represent the predictions of $F_2^{c\bar{c}}$ using the MRST04FF3nlo(red) and CTEQ5F3(blue) parton densities. The shaded band shows the relative change of the central value due to variation of the model parameters: 1.3$< m_c <$1.6 GeV, $0.5\mu < \mu_r = \mu_f < 2\mu$ and using the CTEQ5F3 parton densities. The latter is shown in addition separately by the dark shaded band.

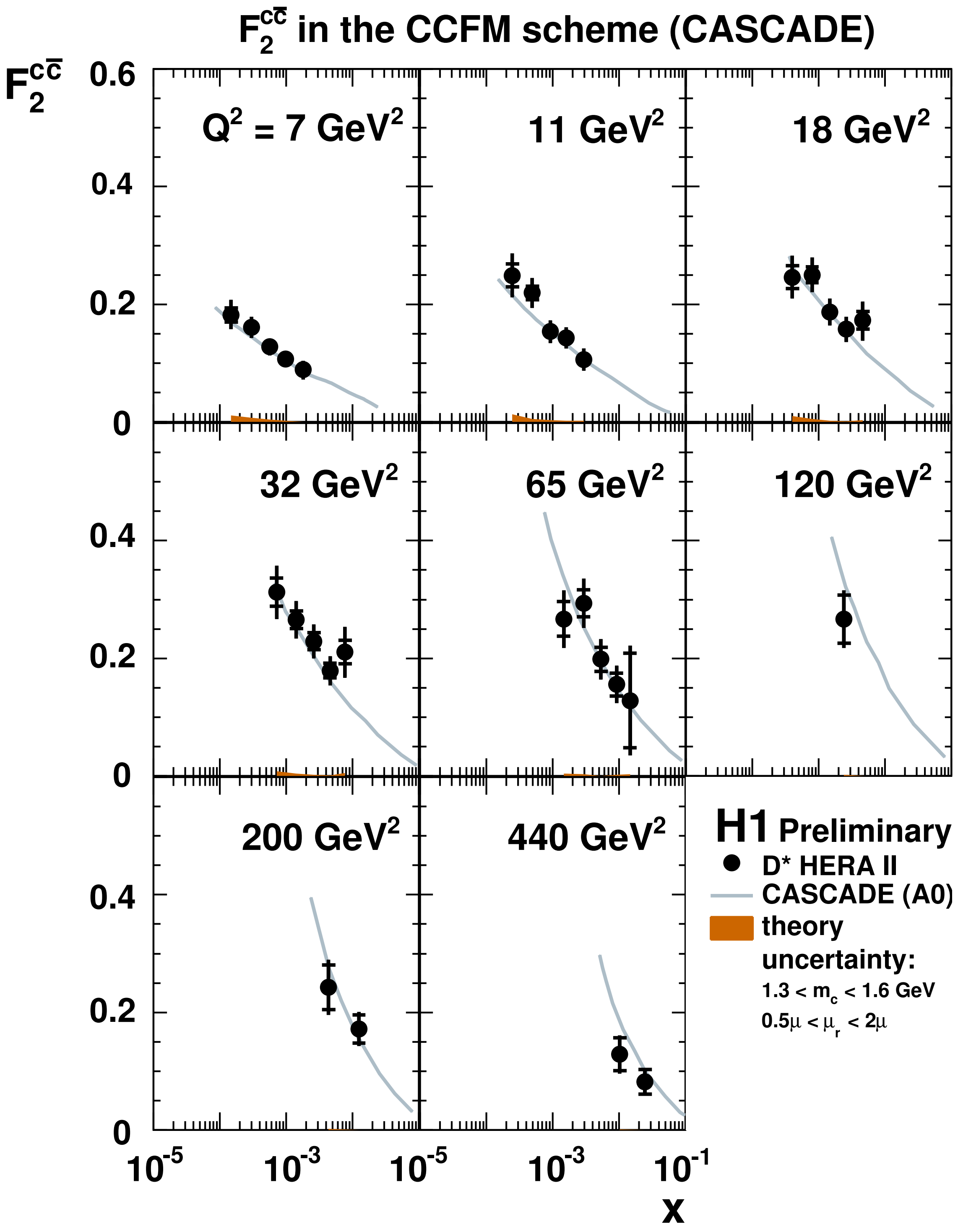

Figure 5: The charm contribution to the proton structure function, $F_2^{c\bar{c}}$, as derived from the inclusive $D^{*\pm}$ meson analysis in the framework of CCFM as implemented in the LO Monte-Carlo program CASCADE, using the set-A0 unintegrated gluon density with charm mass $m_c = 1.43$ GeV and renormalization scale $\mu_r = \sqrt{Q^2 + 4m_c^2}$. The error bars refer to the statistical error (inner) and the total error including the extrapolation uncertainty due to the fragmentation model (outer), respectively. The solid line represents the prediction of $F_2^{c\bar{c}}$ with CASCADE using the parameters mentioned before. The shaded band shows the relative change of the central value due to variation of the charm mass and the renormalization scale.