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A Simultaneous, Direct Determination of α_s and the Gluon Density in the Proton in a QCD Fit to DIS Jet Data

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Abstract

We present the results of a QCD fit to different cross sections measured in deep-inelastic scattering with the H1 detector at HERA. In a combined fit to inclusive DIS cross sections and to dijet and inclusive jet cross sections measured in the Breit frame, we determine the strong coupling constant $\alpha_s(M_Z)$ simultaneously with the gluon and the quark densities in the proton. The result shows a large anticorrelation between $\alpha_s(M_Z)$ and the value of the gluon density at fixed values of the momentum fraction x.

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Introduction

Deep-inelastic lepton-proton scattering (DIS) experiments have played a fundamental role in the understanding of the structure of the proton and in establishing Quantum-Chromodynamics (QCD) as the theory of the strong interaction. The inclusive lepton-proton cross section is directly sensitive to the quark densities in the proton, but gives only indirect information on the gluon content and on the strong coupling constant α_s via scaling violations of the structure functions. Observables which are directly sensitive both to α_s and to the gluon density in the proton are the production rates of events in which the final state contains more than one hard jet (besides the proton remnant). These multi-jet cross sections can be used to test the predictions of perturbative QCD (pQCD) and allow a direct determination of α_s and the gluon density.

The large center-of-mass energy $\sqrt{s}$ of $300\,\mathrm{GeV}$ at HERA allows multi-jet production in DIS to be studied over large regions of phase space. The H1 collaboration has presented preliminary measurements of the dijet cross section [1, 2] and the inclusive jet cross section [3, 4] in the Breit frame at large four-momentum transfers $150 < Q^2 < 5\,000\,\mathrm{GeV}^2$. In a QCD fit to the inclusive jet cross section we have determined α_s as a function of the transverse jet energy E_T in the Breit frame [4] using parton density parameterizations obtained in global fits [5, 6]. A consistent determination of the quark and the gluon densities in the proton has been obtained in a simultaneous fit to the dijet cross section [2] and the inclusive DIS cross section [7] in which $\alpha_s(M_Z)$ has been taken to be the world average value.

In this paper we present a simultaneous determination of α_s, the gluon and the quark densities in the proton from a QCD fit to the dijet and the inclusive jet cross section and to the inclusive DIS cross section. We give a brief description of the assumptions and methods used in the QCD fit and present the result as a correlation plot between $\alpha_s(M_Z)$ and the gluon density at fixed values of x.

Strategy

In perturbative QCD (pQCD) the cross section of any process in deep-inelastic lepton-proton scattering can be written as a convolution of (process specific) perturbative coefficients $c_{a,n}$ with (universal) parton density functions $f_{a/p}$ of the proton

$$\sigma = \sum_{a,n} \int_0^1 \mathrm{d}x \; \alpha_s^n(\mu_r) \; c_{a,n}\left(\frac{x_{\mathrm{Bj}}}{x}, \mu_r, \mu_f\right) \; f_{a/h}(x, \mu_f) \, . \tag{1}$$

The sum runs over all parton flavors a and all orders n considered in the perturbative expansion. The integration is carried out over all fractional parton momenta x. The coefficients $c_{a,n}$ are predicted by pQCD. They are currently known to next-to-leading order in the strong coupling constant for the inclusive DIS cross section ($n = 0, 1$) and for the dijet and the inclusive jet cross section ($n = 1, 2$) [8]. In the regions of sufficiently large transverse jet energies and not too large values of Q^2 ($Q^2 < 5\,000\,\mathrm{GeV}^2$) the effects from quark masses and from Z° exchange can safely be neglected [9]. In the approximation of photon induced processes and for massless quarks, in each order of α_s the perturbative coefficients of the quarks fulfill the relations

$$c_u = c_c = c_t \;=\; c_{\bar{u}} = c_{\bar{c}} = c_{\bar{t}} \qquad \text{and} \qquad c_d = c_s = c_b \;=\; c_{\bar{d}} = c_{\bar{s}} = c_{\bar{b}}\,. \tag{2}$$

Therefore only three coefficients are independent and the cross section can be described by three independent parton density functions[1] $xG(x)$, $x\Delta(x)$ and $x\Sigma(x)$ with coefficients c_G, c_Δ and c_Σ, which have to be defined such that

$$c_g g(x) \,+\, \sum_a c_a q_a(x) \;=\; c_G\, G(x) \,+\, c_\Sigma\, \Sigma(x) \,+\, c_\Delta\, \Delta(x)\,. \tag{3}$$

We chose these three parton density functions as

$$\begin{aligned}
\text{Gluon:} \quad & x\, G(x) \;\equiv\; x\, g(x)\,, \\
\text{Sigma:} \quad & x\, \Sigma(x) \;\equiv\; x \sum_a \left(q_a(x) + \bar{q}_a(x)\right), \\
\text{Delta:} \quad & x\, \Delta(x) \;\equiv\; x \sum_a e_a^2\, \left(q_a(x) + \bar{q}_a(x)\right),
\end{aligned} \tag{4}$$

where the sums run over all quark flavors a and e_a denotes the electrical charge of the corresponding quark. The three corresponding coefficients are given by linear combinations of the single flavor coefficients

$$c_G = c_{\text{gluon}} \qquad c_\Sigma = 1/3\; (4\, c_d - c_u) \qquad c_\Delta = 3\; (c_u - c_d)\,. \tag{5}$$

At orders $\mathcal{O}(\alpha_s^0)$ and $\mathcal{O}(\alpha_s^1)$ the contributions from different quark flavors are proportional to their electrical charge squared (i.e. $c_u = 4\, c_d$). Therefore the coefficient c_Σ in (5) vanishes and the only quark contributions to the cross sections come from $x\Delta(x)$. The gluon gives contributions at order $\mathcal{O}(\alpha_s)$ and higher. $x\Sigma(x)$ starts to contribute at order $\mathcal{O}(\alpha_s^2)$ and does therefore not contribute to the inclusive DIS cross section up to next-to-leading order. Table 1 gives an overview of the orders in which the parton densities contribute to the different processes (up to NLO). $x\Sigma(x)$ enters only the jet cross sections via the next-to-leading order corrections. At large Q^2 these contributions are, however, small (4.5% for the dijet cross section at $150 < Q^2 < 200\,\text{GeV}^2$, decreasing to 2% at $600 < Q^2 < 5000\,\text{GeV}^2$). In the following we use the parameterization CTEQ5M1 to determine this contribution which is not regarded as a degree of freedom in the analysis. This is, however, only a weak assumption which will (due to the smallness of the contribution) not bias any result.

[1] We do not explicitly display the dependence on the factorization scale.

	LO	NLO
$\sigma_{\text{incl. DIS}}$	$x\Delta(x)$	$x\Delta(x),\ xG(x)$
σ_{jets}	$xG(x),\ x\Delta(x)$	$xG(x),\ x\Delta(x),\ x\Sigma(x)$

Table 1: Overview of the orders at which the parton density functions contribute to different cross sections

With this approximation the inclusive DIS cross section and the jet cross section now depend on three quantities which will be determined in this analysis: α_s, the gluon density $xG(x)$ and the quark density $x\Delta(x)$. To demonstrate the basic sensitivity we write the leading order cross sections in the symbolic form

$$\begin{array}{lrcl} \text{inclusive DIS cross section:} & \sigma_{\rm incl.\,DIS} & \propto & \Delta \\ \text{jet cross sections in DIS:} & \sigma_{\rm jet} & \propto & \alpha_s \cdot (c_G\, G \,+\, c_\Delta\, \Delta)\,. \end{array} \tag{6}$$

These relations make clear that in DIS a direct determination of either α_s or the gluon density can never be performed without considering the correlation with the other quantity. The quark densities are directly constrained by the inclusive DIS cross section. Using the jet cross section we can then determine either the gluon density or α_s. For a simultaneous determination of both additional information is needed, e.g. a measurement of the jet cross section in a different kinematic region where the perturbative prediction is given by

$$\sigma'_{\rm jet} \;\propto\; \alpha_s \cdot (c'_g\, g \,+\, c'_\Delta\, \Delta) \qquad \text{with} \qquad \frac{c'_g}{c'_\Delta} \neq \frac{c_g}{c_\Delta}\,. \tag{7}$$

This is in fact the case for the inclusive jet and the dijet cross section in different regions of Q^2, since the gluon induced fraction of the cross section decreases towards higher Q^2.

Fitting Technique

A determination of theoretical parameters can only be performed in phase space regions where theoretical predictions are reliable. We therefore restrict the QCD analysis to the region where NLO corrections to the jet cross sections are reasonably small with k-factors below 1.4. This is the region of $Q^2 > 150\,{\rm GeV}^2$. The QCD analysis is using the inclusive jet cross section presented in [4] and the dijet cross section presented in [2]. We include data on the (reduced) inclusive DIS cross section to exploit their sensitivity to the quark densities in the proton. We use a subsample of our recently published measurement [7] in the range $150 \leq Q^2 \leq 1\,000\,{\rm GeV}^2$. Since the present analysis uses the same experimental techniques the effects of the point to point correlated experimental uncertainties can be fully taken into account.

The fit of the theoretical parameters is performed in a χ^2 minimization using the program MINUIT [10]. The definition of χ^2 [11] fully takes into account all correlations of experimental uncertainties between the data points and the correlations of uncertainties between the theoretical predictions. This χ^2 definition has also been used in recent global data analyses [12, 13] and in a previous H1 publication [14].

In the fitting procedure the perturbative QCD predictions in NLO for the inclusive DIS cross section are directly compared to the data, while the NLO predictions for the jet cross sections are divided by the hadronization corrections before they are compared to the jet data

$$\begin{array}{rcl} \sigma^{\rm H1}_{\rm incl.\,DIS} & \longleftrightarrow & \sigma^{\rm NLO}_{\rm incl.\,DIS} \\ \sigma^{\rm H1}_{\rm jet} & \longleftrightarrow & \sigma^{\rm NLO}_{\rm jet} \cdot c^{-1}_{\rm hadronization} \qquad \text{with} \qquad c_{\rm hadronization} = \dfrac{\sigma^{\rm parton}_{\rm jet}}{\sigma^{\rm hadron}_{\rm jet}}\,. \end{array}$$

The hadronization corrections are determined as described in [2, 4] using the average value from the model predictions by HERWIG, LEPTO and ARIADNE. The uncertainty from the model and the parameter dependence of these predictions is always below 3% [15]. We take the uncertainty in the matching of the parton level (parton cascade and NLO calculation) into account by increasing the quoted uncertainty in those regions where the hadronization corrections are large. In detail, we define the uncertainty of the hadronization correction for each bin of the jet cross sections to be half the size of the correction, but at least 3%. This uncertainty is assumed to be correlated between the theoretical predictions for all data points.

The renormalization scale μ_r^2 in the NLO calculation is identified with the process specific hard scales in both processes. The inclusive DIS cross section is evaluated at $\mu_r^2 = Q^2$ and the jet cross sections are evaluated at $\mu_r^2 = E_T^2$ (the transverse jet energy in the Breit frame). The strong coupling constant $\alpha_s(\mu_r^2)$ is parameterized in terms of its value at the scale $\mu_r^2 = M_Z^2$ using the numerical solution of the renormalization group equation in 4-loop accuracy[2] [16, 17].

In principle, the arguments invoked in the choice of the renormalization scale μ_r also apply to the factorization scale μ_f for the inclusive DIS cross section and for the jet cross section. However, we make a different choice for the following reasons. We have combined the different parton flavors into three independent parton density functions $xG(x, \mu_f)$, $x\Delta(x, \mu_f)$ and $x\Sigma(x, \mu_f)$. These three parton densities are, however, only independent as long as no evolution between different scales μ_f is performed. The evolution of the gluon density is coupled to the evolution of $x\Sigma(x, \mu_f)$. Furthermore, since $x\Delta(x, \mu_f)$ is not an eigenstate of the DGLAP evolution operators the evolution requires its decomposition into a non-singlet and a singlet (i.e. $x\Sigma(x, \mu_f)$). This introduces an additional dependence between the quark densities. To avoid mixing between the different parton densities we do not evolve the parton distributions to different scales but perform the perturbative calculations at a fixed value of the factorization scale $\mu_f^2 = \mu_0^2$. The jet cross sections are sensitive to the parton distributions in the x-range of $0.008 \lesssim x \lesssim 0.3$. In this x-range the factorization scale dependence of the parton density functions is small. In a next-to-leading order calculation the remaining μ_f^2 dependence given by the DGLAP evolution equations is largely compensated by a corresponding term in the perturbative coefficients. The perturbative cross sections therefore depend only weakly on the choice of the factorization scale. The difference between using a fixed factorization scale μ_0^2 and performing the full DGLAP evolution at a scale μ_f^2 is of higher order in α_s.

If the scale μ_f^2 is close to the fixed scale μ_0^2 (such that $\ln(\mu_f^2/\mu_0^2) \lesssim 1$) these higher order terms are small. We therefore decide to use a fixed value of the factorization scale of the order of the average transverse jet energies in the dijet and the inclusive jet cross section $\mu_f^2 = \mu_0^2 = 200\,\mathrm{GeV}^2 \simeq \langle E_T^2 \rangle$. The subsample of the (reduced) inclusive DIS cross section $150 \leq Q^2 \leq 1\,000\,\mathrm{GeV}^2$ has been chosen such that the four-momentum transfer squared is also approximately of the same size $Q^2 \simeq \mu_0^2 = 200\,\mathrm{GeV}^2$.

The renormalization and the factorization scale dependence of the cross sections is considered as a correlated theoretical uncertainty. Both scales are (separately) varied by a factor x_μ around their nominal values μ_0^2 in the range $x_\mu = \{\frac{1}{4}, 4\}$ and the ratios $\frac{\sigma_{\mathrm{NLO}}(x_\mu \cdot \mu_0^2)}{\sigma_{\mathrm{NLO}}(\mu_0^2)}$ are taken as the corresponding uncertainties. Together with the uncertainty from the hadronization corrections they constitute the quoted theoretical uncertainty of the fit result.

[2]It has been checked that in the range of scales considered in this analysis $7\,\mathrm{GeV} < \mu_r < M_Z$ the differences between the 2-, 3- and 4-loop solutions are always below 3 per mil.

During the χ^2 minimization procedure in the fit the NLO calculations of the jet cross sections have to be performed iteratively for different values of $\alpha_s(M_Z)$ and for different parton density functions (the number of calculations used to obtain the present results and to study their stability is in the order of one million). Since standard computations of NLO jet cross sections are time consuming we use the method [9] of pre-convoluting the perturbative coefficients with suitably defined functions which can then be folded with the parton densities and α_s for the fast computation of the NLO cross section. The perturbative coefficients are calculated using the the program DISENT [18]. The NLO calculations are performed in the $\overline{\mathrm{MS}}$ scheme for $n_f = 5$.

QCD Fit

A complete description of jet production in DIS in terms of our present quantitative level of understanding of hadronic physics, namely NLO pQCD, is only obtained if the cross sections can be used to determine simultaneously *both* the appropriate QCD parameters (in this case α_s) *and* the inherently incalculable proton pdfs. We have attempted such a determination, and thereby demonstrated the extent to which such an approach can be used. We perform a simultaneous fit of the parton densities and $\alpha_s(M_Z)$ using the inclusive DIS cross section, the inclusive jet cross section $\mathrm{d}^2\sigma_{\mathrm{jet}}/\mathrm{d}E_T\mathrm{d}Q^2$ and the dijet cross section $\mathrm{d}^2\sigma_{\mathrm{dijet}}/\mathrm{d}\xi\mathrm{d}Q^2$ (both measured with the inclusive $k_\perp$ algorithm). The gluon and the quark distributions are parameterized by

$$xP(x) = A\; x^b\; (1-x)^c\; (1+dx) \tag{8}$$

where $xP(x)$ stands for $xG(x)$ or $x\Delta(x)$. The simultaneous fit yields $\chi^2/\mathrm{n.d.f.} = 61.10/104$ and a result for the quark distributions identical to the one obtained in the fit of the gluon density with a constrained $\alpha_s(M_Z)$ (presented in [2]). The results of this simultaneous fit are displayed in Fig. 1 as a correlation plot between $\alpha_s(M_Z)$ and the gluon density evaluated at four different values of $x = 0.01,\ 0.02,\ 0,04,\ 0.1$ which lie in the range where the jet cross sections are sensitive. The central fit result is indicated by the full marker and the error ellipse is the contour along which the χ^2 of the fit is by one larger than the minimum (including experimental and theoretical uncertainties). The ellipticity of the contours indicate that our data are very sensitive to the product $\alpha_s \cdot xg(x)$ but do not yet allow a determination of both parameters simultaneously with useful precision.

Also included in Fig. 1 are the results from global fits. All of these results are within the contour except for GRV98 [19] (at $x < 0.04$) which use a relatively small value of $\alpha_s(M_Z) = 0.114$.

The stability of the results in Fig. 1 has been tested by using different parameterizations of the parton density functions (involving five parameters). Fits have been performed excluding either the low ($Q^2 < 200\,\mathrm{GeV}^2$) or the high ($Q^2 > 600\,\mathrm{GeV}^2$) Q^2 data. Although the fits give consistent central results, the high Q^2 data is needed to make a stable determination of the contour of the ellipsoid. This indicates that the data at high Q^2 are of special importance in this combined fit, due to their smaller theoretical uncertainties.

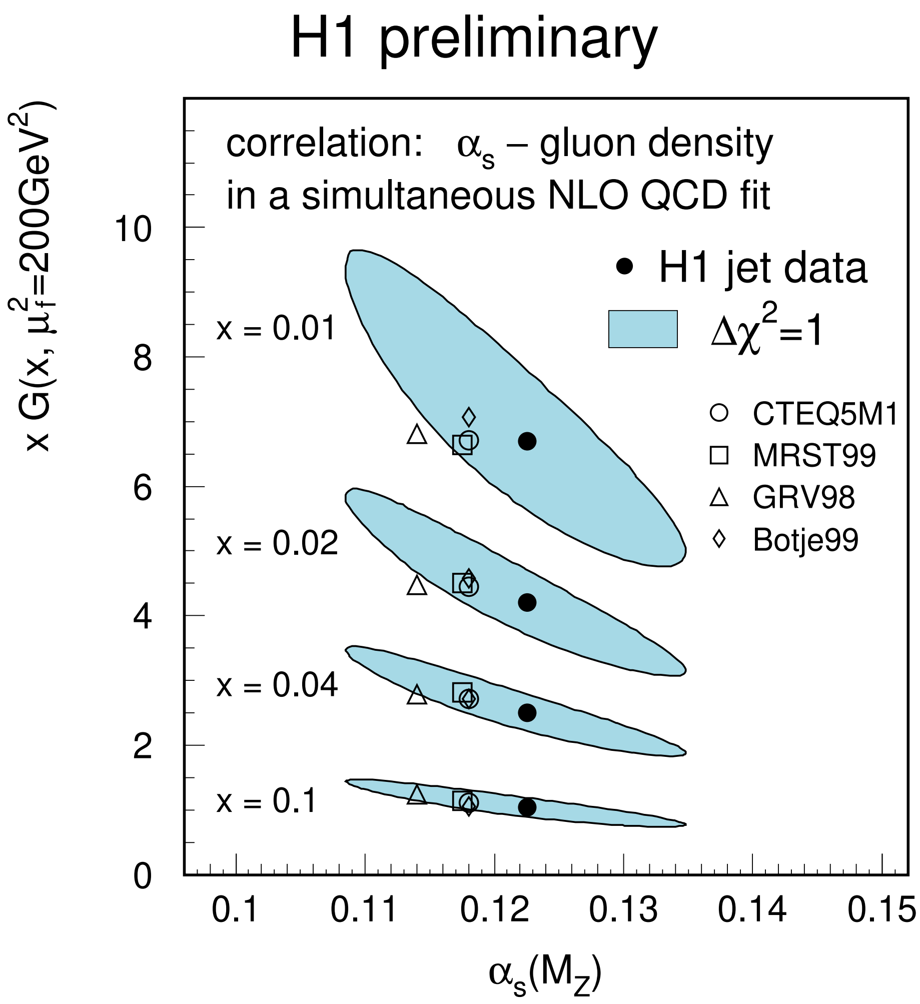

Figure 1: The correlation of the fit results for $\alpha_s(M_Z)$ and the gluon density at four different values of x, determined in a simultaneous QCD fit to the inclusive DIS cross section, the inclusive jet cross section and the dijet cross section. The jet cross sections are measured using the inclusive $k_\perp$ jet algorithm. The error ellipses include the experimental and the theoretical uncertainties.

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