A New H1 NLO QCD Fit

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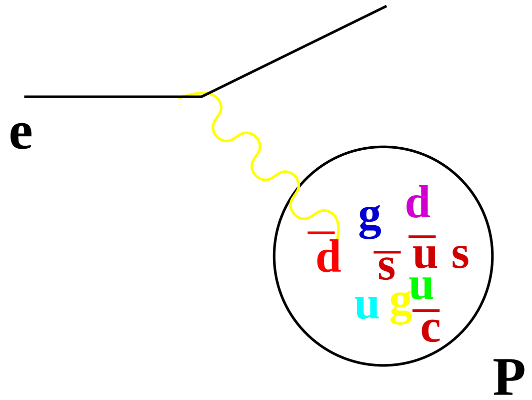
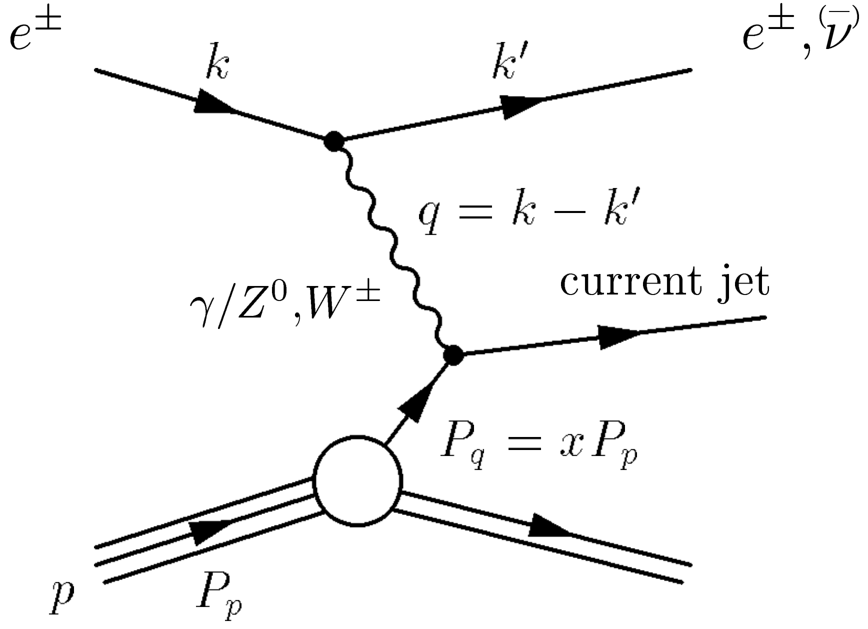


Deep Inelastic Scattering



4-momentum transfer
$Q^2 = -q^2 = -(k - k')^2$
parton momentum fraction
$x = Q^2/(2p \cdot q)$
inelasticity
$y = p \cdot q/(p \cdot k)$
center of mass (cms) energy $\sqrt{s}$:
$s = (k + p)^2$

at fixed cms energy: $y = Q^2/xs$

neutral current (NC):
γ, Z^0 exchange

charged current (CC):
$W^\pm$ exchange

lepton vertex: pointlike particle,
determined by electroweak Standard Model (SM)
proton vertex: object with structure

quark-parton-modell (QPM):
elastic scattering on pointlike parton (quark);
quark momentum distribution $xq(x)$ inside proton

NC Cross Section and Structure Functions

Inclusive NC Cross Section (γ^*, Z^0 exchange)

$$\frac{d^2\sigma^{e^\mp p}}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left(Y_+\tilde{F}_2(x,Q^2) \pm Y_- x\tilde{F}_3(x,Q^2) - y^2 F_L(x,Q^2)\right),$$

$$Y_\pm(y) = 1 \pm (1-y)^2$$

Generalized Structure Funktionen Quark Parton Model:

$$\tilde{F}_2 = \sum_i A_i(Q^2)\,[xq_i + x\bar{q}_i]$$

$$x\tilde{F}_3 = \sum_i B_i(Q^2)\,[xq_i - x\bar{q}_i]$$

Electroweak Coefficient Functions

$A_i(Q^2)$, $B_i(Q^2)$ (QED: $A_i = e_i^2$, $B_i = 0$)

$$F_L = 0 \text{ in LO } (\neq 0 \text{ in QCD })$$

$$F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{\mathrm{d}z}{z^3}\left[\frac{16}{3}\sum_q z e_q^2 (q+\bar{q}) + 8\sum_q e_q^2 \left(1-\frac{x}{z}\right)\cdot zg\right]$$

Parton Density Functions (PDFs)

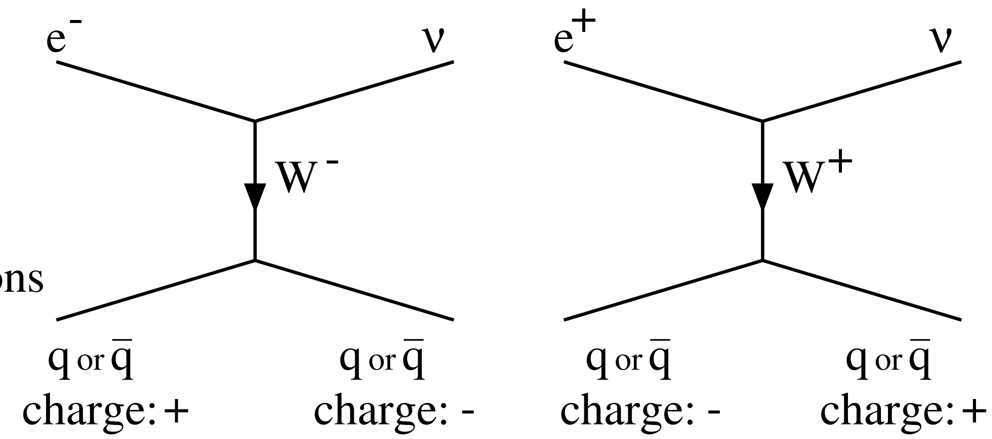
$g = g(x,Q^2$, $q_i = q_i(x,Q^2)$, $\bar{q}_i = \bar{q}_i(x,Q^2)$, $i = u,d,s,c,b$

CC Cross Sections and Strukture Functions

W bosons couple to

left handed fermions

right handed anti fermions



$$\frac{d^2\sigma_{CC}}{dx\,dQ^2} = \frac{G_F^2}{2\pi x}\left(\frac{{M_W}^2}{{M_W}^2 + Q^2}\right)^2 \cdot \Phi_{CC}^{e^\pm p}(x, Q^2)$$

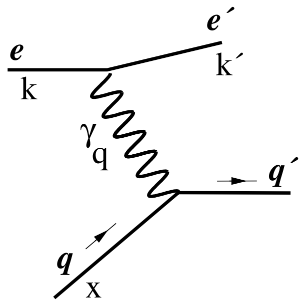
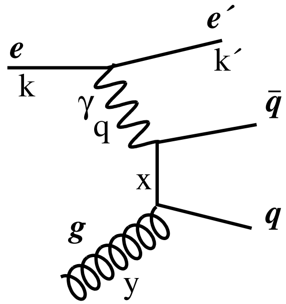
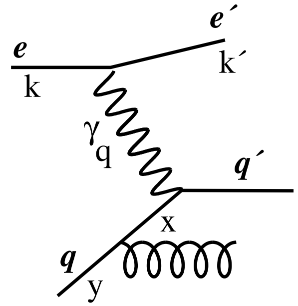
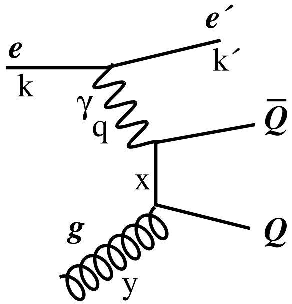
$$\Phi_{CC}^{e^- p}|_{LO} = (u + c) + (1 - y)^2(\bar{d} + \bar{s})$$
$$\Phi_{CC}^{e^+ p}|_{LO} = (\bar{u} + \bar{c}) + (1 - y)^2(d + s)$$

G_F Fermi coupling constant, M_W mass of $W^\pm$ Boson

Parton Evolution

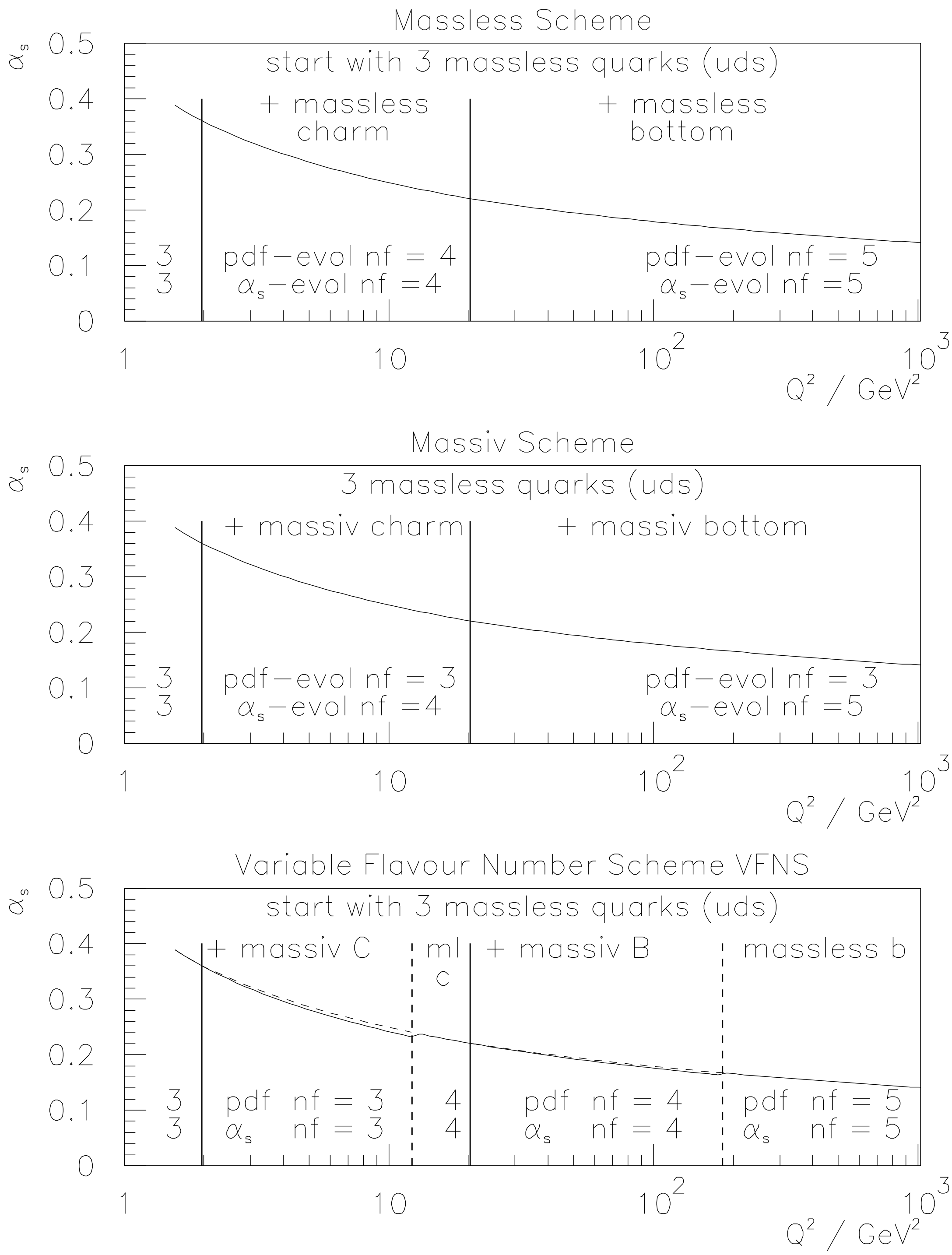
$PDF = f(x, Q^2)$ no prediction of x-dependence

Q^2-dependence predicted by QCD

	LO	**NLO**
3, 4, 5 massless ***q* = *u d s* (*c*, *b*)**		 
3, 4 + 2, 1 massiv ***Q* = *c* (*b*)**		

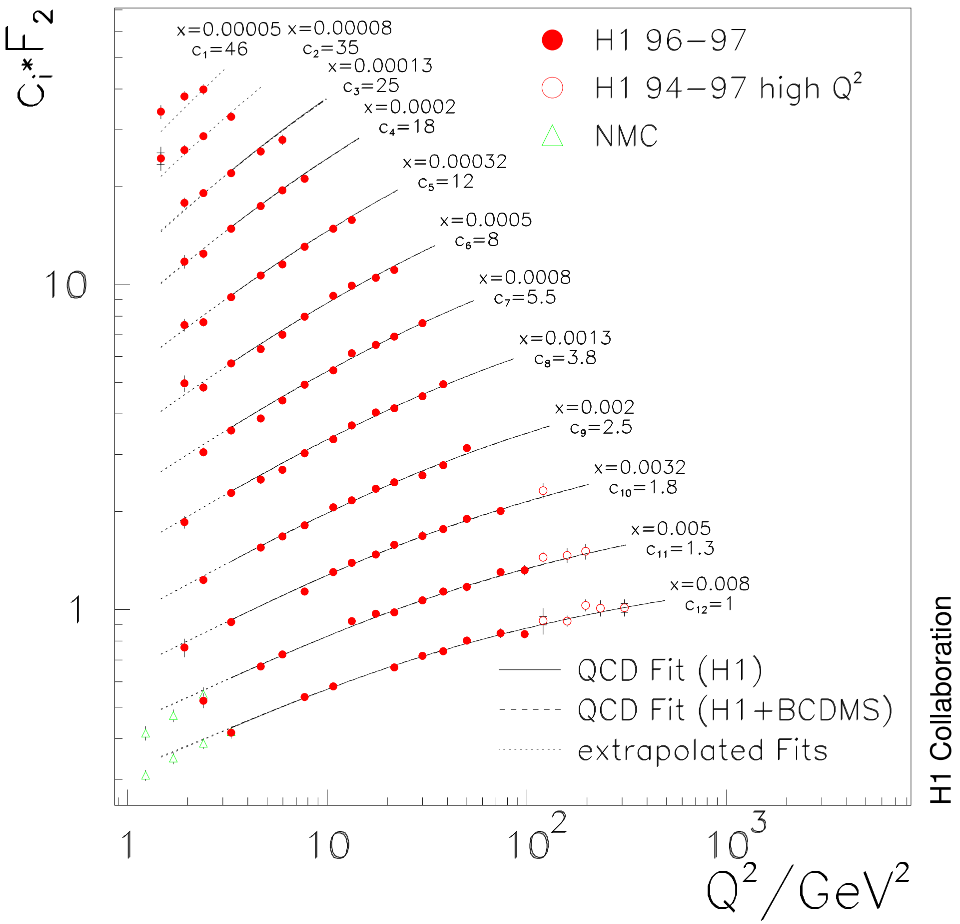
Q^2 evolution described by DGLAP equations

Choice of NLO Scheme



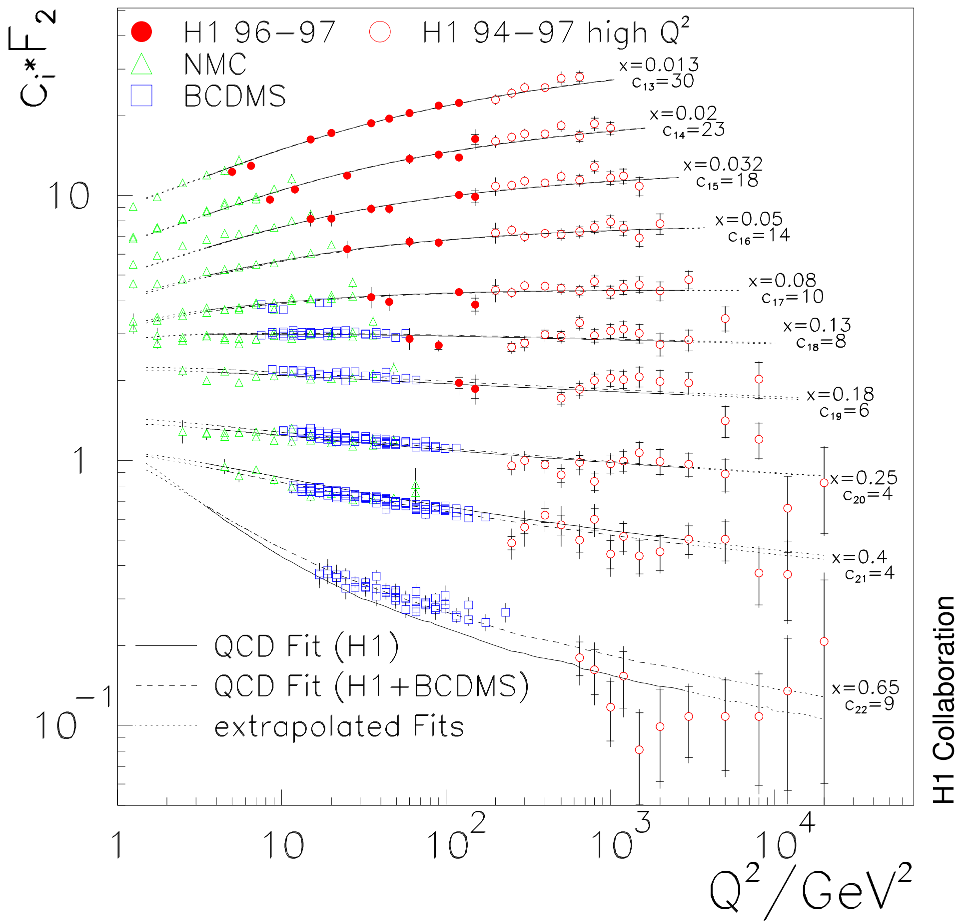
Fit is performed in NLO massless scheme.

F_2 Measurement, Scaling Violations



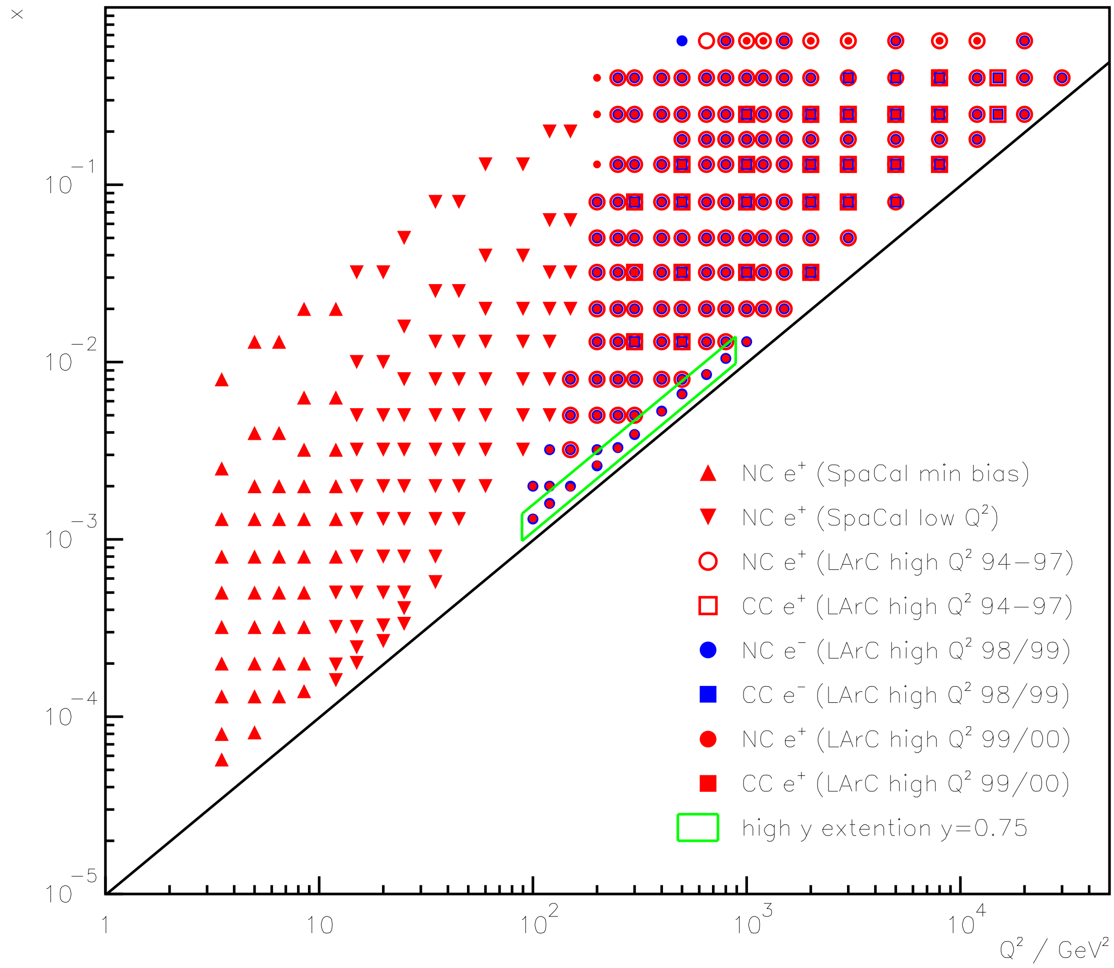
H1 96-97 precision: stat $\leq$ **1%**

syst $\leq$ **3%**



scaling violations well described by the NLO QCD fit

H1 Inclusive Data



- Question: How do H1 data alone constrain PDFs?
- New measurements at high $y = 0.75$,
  sensitivity to F_L at high Q^2,
  extraction of F_L requires prediction for F_2.

Definition of χ^2

"Pascaud and Zomer" method, extended χ^2 (preprint LAL-95-05):

$$\chi^2 = \sum_k^{n_{exp}} \sum_i^{N_k} \frac{\left(\sigma_{k,i}^{exp} - \sigma_{k,i}^{th} \cdot (1 - n_k \delta_k^{norm} - \sum_j^{n_s(k)} s_{k,j}\, \delta_{k,i,j}^{syst})\right)^2}{\delta_{k,i}^{sta\,2} + \delta_{k,i}^{unc\,2}}$$

$$+ \sum_k^{n_{exp}} n_k^2 + \sum_k^{n_{exp}} \sum_j^{n_{s(k)}} s_{j,k}^2$$

$\sigma_{k,i}^{exp}$: measurement i of dataset k
$\sigma_{k,i}^{th}$: prediction for data point i of dataset k
n_{exp}: number of datasets
N_k: number of data points in dataset k

$\delta_{k,i}^{sta}$: statistical error
$\delta_{k,i}^{unc}$: uncorrelated systematic error
$\delta_{k,i,j}^{sys}$: correlated systematic error of source j
δ_k^{norm}: uncertainty of dataset normalisation
n_k: shift of normalisation
$s_{k,j}$: shift of systematic of source j

data central values can be moved by shifts of n_k, $s_{k,j}$
shifts cost an additional penalty of n_k^2, $s_{k,j}^2$ to χ^2
n_k, $s_{k,j}$ are determined by the fit
correlation of systematic sources between datasets taken into account

Error Bands

From the fit (using MINUIT) we obtain
– a set of parameters $p_1,\ p_2, \ldots,\ p_n$ characterising the input parton densities at starting scale
– a set of dataset normalisation factors and shifts due to correlated systematics
– information on the parameter errors
(χ^2 second derivative matrix, HESSE algorithm)

To obtain the error bands on PDFs, structure fuctions and cross sections
for a given set of kinematic variables x, Q^2, y, s the fitted paramters are shifted
under the constraint $\boldsymbol{\chi^2 = \chi^2_{min} + 1}$.

The error band will depend upon
– the paramterisation of the input parton densities
– Q_0^2 as this implicitly hanges the parameterisation at Q^2
– the error band will depend upon the assuptions which enter the fit

$\mathcal{O}\ 10^5$ fits are performed where all essential ingredients to the fit are varied (α_s, m_c, m_b, ...)

Parameterisation of PDFs

general form:
$pdf(x,Q_0^2) = A_{pdf}\, x^{B_{pdf}} (1-x)^{C_{pdf}} (1 + a_{pdf}\sqrt{x} + b_{pdf}x)$

$$\begin{array}{lrcl}
\text{TYPE 3}: & pdf(x,Q_0^2) & = & A_g x^{B_g}(1-x)^{C_g} \cdot 1 \\
\text{TYPE 4}: & pdf(x,Q_0^2) & = & A_g x^{B_g}(1-x)^{C_g} \cdot (1 + a\sqrt{x}) \\
\text{TYPE 5}: & pdf(x,Q_0^2) & = & A_g x^{B_g}(1-x)^{C_g} \cdot (1 \qquad\quad + bx) \\
\text{TYPE 6}: & pdf(x,Q_0^2) & = & A_g x^{B_g}(1-x)^{C_g} \cdot (1 + a\sqrt{x} + bx)
\end{array}$$

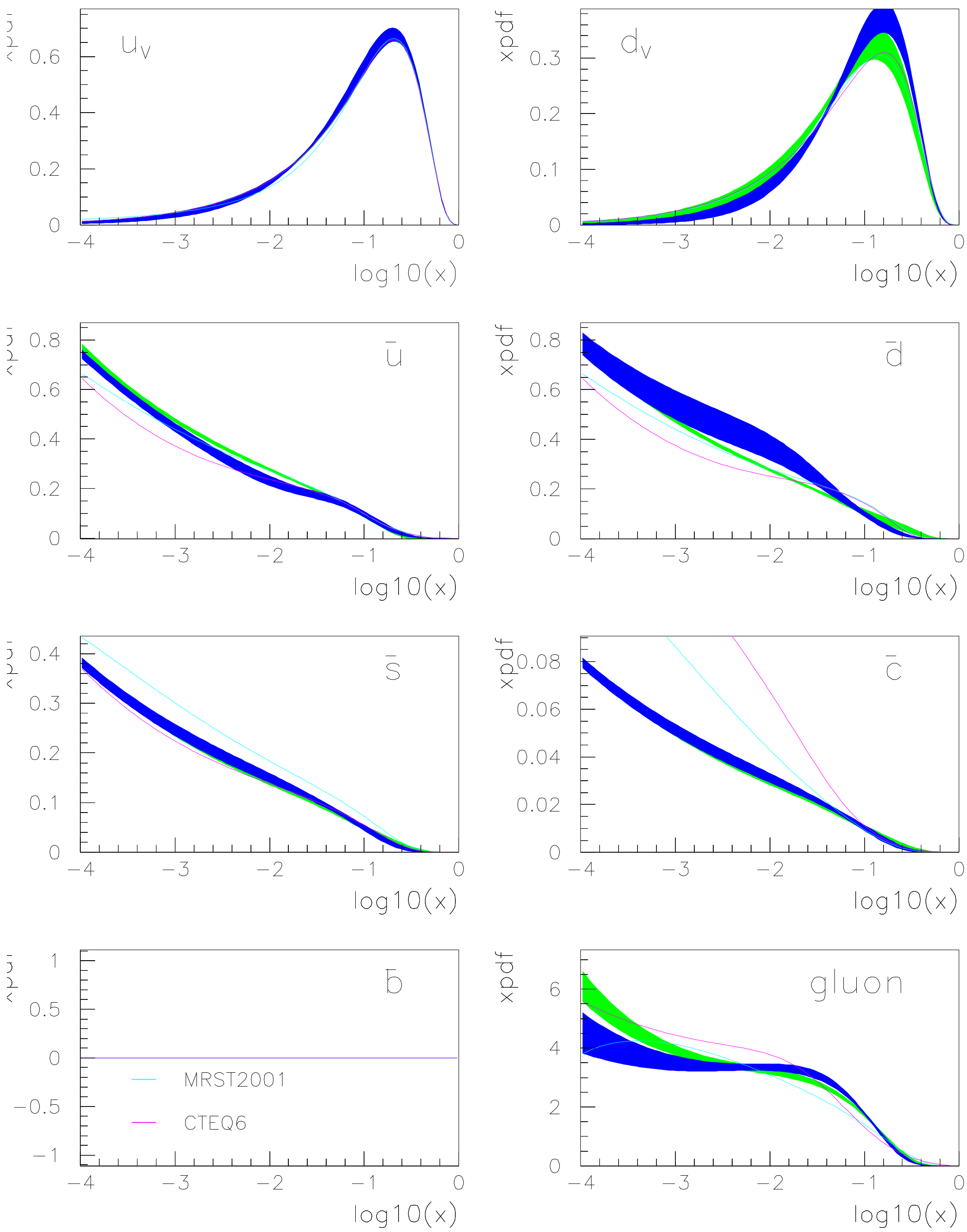
PDFs at starting scale:
$g,\ u_{val},\ d_{val},\ \bar{u},\ \bar{d}-\bar{u}$
$s+\bar{s} = (\frac{1}{2}+\epsilon)(\bar{u}+\bar{d})$, $\bar{c} =$ fraction of $\bar{u}$ if $Q_o^2 > m_c^2$; $b = 0$

Constraints:

u counting:
$u_{val} = \int_0^1 (u(x,Q^2) - \bar{u}(x,Q^2))\mathrm{d}x = 2$
d counting:
$d_{val} = \int_0^1 (d(x,Q^2) - \bar{d}(x,Q^2))\mathrm{d}x = 1$
momentum sum rule:
$p_p = \int_0^1 xg(x,Q^2) + \sum_q (xq(x,Q^2) + x\bar{q}(x,Q^2))\mathrm{d}x = 1$
sea quarks:
$\int_0^1 (\bar{d}-\bar{u})\mathrm{d}x = 0.118$
$\bar{d}/\bar{u} \to 1 + ax$ for $x \to 0$: $B_{\bar{d}-\bar{u}} = B_{\bar{d}} + 1$
Best Fit: H1 56333 (H1+BCDMS 56633)

Example PDFs

Variation of Parametric Form (45533 vs 45553)



error bands experimental errors only

Variation of Input Parameters

Parameter	Central	Variation	Comment
Q^2_{min}	3.5 GeV^2	(2-5)	as for low Q^2 Fit
Q^2_o	4.0 GeV^2	(2.5-6.0)	as for low Q^2 Fit
mass_c	1.4 GeV	± 0.2	uncertainty of definition
mass_b	4.5 GeV	± 0.5	uncertainty of definition
α_s	0.1185	± 0.0020	world average
$\int(\bar{d}-\bar{u})\,\mathrm{d}x$	0.118	(0.075-0.151)	fixed, free within errors
$\bar{c}$-fraction of $\bar{u}$	10%	$\pm 5\%$	i.e. $4 \pm 2\%$ of sea
y-cut	0.35	(0.3-0.5)	F_L extraction only

other fixed inputs:

M_W	80.41	fixed
M_Z	91.187	fixed
$\sin^2\theta_w$	0.232	fixed
Δ_r (ew rad cor)	0.0349	fixed
target mass correction	0.88	fixed
μ_f	1.00	fixed
μ_r	1.00	fixed

variation of data sets
variation of parametric form

in total $\mathcal{O}$ 10^5 fits

Summary

The concepts of a new fit to H1 data have been presented

- exploit full set of H1 inclusive measurements
- use H1 data alone as far as possible
- include correlated systematic errors in the fit
- allow central values of measurements to be moved
- experimantal errorbands ($\Delta\chi^2 = 1$)
- quantify model uncertainties by variation of fit assumptions