DESY-01-104
July 2001

ISSN 0418-9833

On the Rise of the Proton Structure Function F_2 Towards Low x

H1 Collaboration

Abstract:

A measurement of the derivative $(\partial \ln F_2/\partial \ln x)_{Q^2} \equiv -\lambda(x, Q^2)$ of the proton structure function F_2 is presented in the low x domain of deeply inelastic positron–proton scattering. For $5 \cdot 10^{-5} \leq x \leq 0.01$ and $Q^2 \geq 1.5\,\mathrm{GeV}^2$, $\lambda(x, Q^2)$ is found to be independent of x and to increase linearly with $\ln Q^2$.

Submitted to Phys. Lett. B

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Dixon[19], V. Dodonov[12], J.D. Dowell[3], A. Droutskoi[23], A. Dubak[25], C. Duprel[2], G. Eckerlin[10], D. Eckstein[35], V. Efremenko[23], S. Egli[32], R. Eichler[36], F. Eisele[13], E. Eisenhandler[19], M. Ellerbrock[13], E. Elsen[10], M. Erdmann[10,40,e], W. Erdmann[36], P.J.W. Faulkner[3], L. Favart[4], A. Fedotov[23], R. Felst[10], J. Ferencei[10], S. Ferron[27], M. Fleischer[10], Y.H. Fleming[3], G. Flügge[2], A. Fomenko[24], I. Foresti[37], J. Formánek[30], G. Franke[10], E. Gabathuler[18], K. Gabathuler[32], J. Garvey[3], J. Gassner[32], J. Gayler[10], R. Gerhards[10], C. Gerlich[13], S. Ghazaryan[4,34], L. Goerlich[6], N. Gogitidze[24], M. Goldberg[28], C. Grab[36], H. Grässler[2], T. Greenshaw[18], G. Grindhammer[25], T. Hadig[13], D. Haidt[10], L. Hajduk[6], J. Haller[13], W.J. Haynes[5], B. Heinemann[18], G. Heinzelmann[11], R.C.W. Henderson[17], S. Hengstmann[37], H. Henschel[35], R. Heremans[4], G. Herrera[7,44], I. Herynek[29], M. Hildebrandt[37], M. Hilgers[36], K.H. Hiller[35], J. Hladký[29], P. Höting[2], D. Hoffmann[22], R. Horisberger[32], S. Hurling[10], M. Ibbotson[21], Ç. İşsever[7], M. Jacquet[26], M. Jaffre[26], L. Janauschek[25], X. Janssen[4], V. Jemanov[11], L. Jönsson[20], C. Johnson[3], D.P. Johnson[4], M.A.S. Jones[18], H. Jung[20,10], D. Kant[19], M. Kapichine[8], M. Karlsson[20], O. Karschnick[11], F. Keil[14], N. Keller[37], J. Kennedy[18], I.R. Kenyon[3], S. Kermiche[22], C. Kiesling[25], P. Kjellberg[20], M. Klein[35], C. Kleinwort[10], T. Kluge[1], G. Knies[10], B. Koblitz[25], S.D. Kolya[21], V. Korbel[10], P. Kostka[35], S.K. Kotelnikov[24], R. Koutouev[12], A. Koutov[8], H. Krehbiel[10], J. Kroseberg[37], K. Krüger[10], A. Küpper[33], T. Kuhr[11], T. Kurča[16], R. Lahmann[10], D. Lamb[3], M.P.J. Landon[19], W. Lange[35], T. Laštovička[30,35], P. Laycock[18], E. Lebailly[26], A. Lebedev[24], B. Leißner[1], R. Lemrani[10], V. Lendermann[7], S. Levonian[10], M. Lindstroem[20], B. List[36], E. Lobodzinska[10,6], B. Lobodzinski[6,10], A. Loginov[23], N. Loktionova[24], V. Lubimov[23], S. Lüders[36], D. Lüke[7,10], L. Lytkin[12], H. Mahlke-Krüger[10], N. Malden[21], E. Malinovski[24], I. Malinovski[24], R. Maraček[25], P. Marage[4], J. Marks[13], R. Marshall[21], H.-U. Martyn[1], J. Martyniak[6], S.J. Maxfield[18], D. Meer[36], A. Mehta[18], K. Meier[14], A.B. Meyer[11], H. Meyer[33], J. Meyer[10], P.-O. Meyer[2], S. Mikocki[6], D. Milstead[18], T. Mkrtchyan[34], R. Mohr[25], S. Mohrdieck[11], M.N. Mondragon[7], F. Moreau[27], A. Morozov[8], J.V. Morris[5], K. Müller[37], P. Murín[16,42], V. Nagovizin[23], B. Naroska[11], J. Naumann[7], Th. Naumann[35], G. Nellen[25], P.R. Newman[3], T.C. Nicholls[5], F. Niebergall[11], C. Niebuhr[10], O. Nix[14], G. Nowak[6], J.E. Olsson[10], D. Ozerov[23], V. Panassik[8], C. Pascaud[26], G.D. Patel[18], M. Peez[22], E. Perez[9], J.P. Phillips[18], D. Pitzl[10], R. Pöschl[26], I. Potachnikova[12], B. Povh[12], K. Rabbertz[1], G. Rädel[1], J. 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[a] *Supported by the Bundesministerium für Bildung und Forschung, FRG, under contract numbers 05 H1 1GUA /1, 05 H1 1PAA /1, 05 H1 1PAB /9, 05 H1 1PEA /6, 05 H1 1VHA /7 and 05 H1 1VHB /5*
[b] *Supported by the UK Particle Physics and Astronomy Research Council, and formerly by the UK Science and Engineering Research Council*
[c] *Supported by FNRS-NFWO, IISN-IIKW*
[d] *Partially Supported by the Polish State Committee for Scientific Research, grant no. 2P0310318 and SPUB/DESY/P03/DZ-1/99, and by the German Federal Ministry of Education and Research (BMBF)*
[e] *Supported by the Deutsche Forschungsgemeinschaft*
[f] *Supported by VEGA SR grant no. 2/1169/2001*
[g] *Supported by the Swedish Natural Science Research Council*
[h] *Supported by Russian Foundation for Basic Research grant no. 96-02-00019*
[i] *Supported by the Ministry of Education of the Czech Republic under the projects INGO-LA116/2000 and LN00A006, by GA AVČR grant no B1010005 and by GAUK grant no 173/2000*
[j] *Supported by the Swiss National Science Foundation*
[k] *Supported by CONACyT*
[l] *Partially Supported by Russian Foundation for Basic Research, grant no. 00-15-96584*

The inclusive cross section for deeply inelastic lepton-proton scattering is governed by the proton structure function $F_2(x, Q^2)$. Because of the large centre-of-mass energy squared, $s \simeq 10^5$ GeV^2, the ep collider HERA has accessed the region of low Bjorken x, $x > Q^2/s > 10^{-5}$, for four-momentum transfers squared $Q^2 > 1$ GeV^2. One of the first observations at HERA was of a substantial rise of F_2 with decreasing x [1]. However, this rise may be limited at very low x by unitarity constraints.

Perturbative Quantum Chromodynamics (QCD) provides a rigorous and successful theoretical description of the Q^2 dependence of $F_2(x, Q^2)$ in deeply inelastic scattering. In the double asymptotic limit, the DGLAP evolution equations [2] can be solved [3] and F_2 is expected to rise approximately as a power of x towards low x. A power behaviour is also predicted in BFKL theory [4]. The rise is expected eventually to be limited by gluon self interactions in the nucleon [5].

Recently the H1 Collaboration has presented [6] a new measurement of $F_2(x, Q^2)$ in the kinematic range $3 \cdot 10^{-5} \leq x \leq 0.2$ and $1.5 \leq Q^2 \leq 150$ GeV^2 based on data taken in the years 1996/97 with a positron beam energy $E_e = 27.6$ GeV and a proton beam energy $E_p = 820$ GeV. The high accuracy of these data allows the derivative

$$\left(\frac{\partial \ln F_2(x, Q^2)}{\partial \ln x}\right)_{Q^2} \equiv -\lambda(x, Q^2) \tag{1}$$

to be measured as a function both of Q^2 and of x for the first time. Use of this quantity for investigating the behaviour of F_2 at low x was suggested in [7].

Here results are presented of a measurement of this derivative in the full kinematic range available. Data points at adjacent values of x and at fixed Q^2 are used [6] taking account of the full error correlations and the spacing between the x values. The results[1] obtained are presented in Table 1. The sensitivity of the derivative to the uncertainty of the structure function F_L [6] throughout the measured kinematic range is estimated to be much smaller than the total systematic error at the lowest values of x and is negligible elsewhere.

As can be seen in Figure 1, the derivative $\lambda(x, Q^2)$ is independent of x for $x \lesssim 0.01$ to within the experimental accuracy. This implies that the x dependence of F_2 at low x is consistent with a power law, $F_2 \propto x^{-\lambda}$, for fixed Q^2, and that the rise of F_2 , i.e. $(\partial F_2/\partial x)_{Q^2}$, is proportional to F_2/x. There is no experimental evidence that this behaviour changes in the measured kinematic range.

The derivative is well described by the NLO QCD fit to the H1 cross-section data [6], see Figure 1. In DGLAP QCD, for $Q^2 > 3$ GeV^2, the low x behaviour is driven solely by the gluon field, since quark contributions to the scaling violations of F_2 are negligible. At larger x the transition to the valence-quark region causes a strong dependence of λ on x as indicated by the QCD curves in Figure 1.

Figure 2 shows the measured derivative as a function of Q^2 for different x values. The derivative is observed to rise approximately logarithmically with Q^2. It can be represented by a function $\lambda(Q^2)$ which is independent of x within the experimental accuracy.

[1] Note that derivatives at adjacent x values are thus anti-correlated. The data points at $Q^2 = 150$ GeV^2 are obtained from the H1 measurement [8].

The function $\lambda(Q^2)$ is determined from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data, restricted to the region $x \leq 0.01$. The results for $c(Q^2)$ and $\lambda(Q^2)$ are presented in Table 2 and shown in Figure 3. The coefficients $c(Q^2)$ are approximately independent of Q^2 with a mean value of 0.18. As can be seen, $\lambda(Q^2)$ rises approximately linearly with $\ln Q^2$. This dependence can be represented as $\lambda(Q^2) = a \cdot \ln[Q^2/\Lambda^2]$, see Figure 3. The coefficients are $a = 0.0481 \pm 0.0013(\mathrm{stat}) \pm 0.0037(\mathrm{syst})$ and $\Lambda = 292 \pm 20(\mathrm{stat}) \pm 51(\mathrm{syst})$ MeV, obtained for $Q^2 \geq 3.5$ GeV^2. The values of $\lambda(Q^2)$ are more accurate than data hitherto published by the H1 [9] and ZEUS [10] Collaborations.

Below the deeply inelastic region, for fixed $Q^2 < 1$ GeV^2, the simplest Regge phenomenology predicts that $F_2(x, Q^2) \propto x^{-\lambda}$ where $\lambda = \alpha_{I\!P}(0) - 1 \simeq 0.08$ is given by the Pomeron intercept independently of x and Q^2 [11]. When extrapolating the function $\lambda(Q^2)$ into the lower Q^2 region it has the value of 0.08 at $Q^2 = 0.45$ GeV^2, see also [10].

To summarise, the derivative $(\partial \ln F_2/\partial \ln x)_{Q^2}$ is measured as a function both of x and of Q^2 and is observed to be independent of Bjorken x for $x \lesssim 0.01$ and Q^2 between 1.5 and 150 GeV^2. Thus the behaviour of F_2 at low x is consistent with a dependence $F_2(x, Q^2) = c(Q^2)\, x^{-\lambda(Q^2)}$ throughout that region. At low x, the exponent λ is observed to rise linearly with $\ln Q^2$ and the coefficient c is independent of Q^2 to within the experimental accuracy. There is no sign that this behaviour changes within the kinematic range of deeply inelastic scattering explored.

Acknowledgements We are very grateful to the HERA machine group whose outstanding efforts made this experiment possible. We acknowledge the support of the DESY technical staff. We appreciate the substantial effort of the engineers and technicians who constructed and maintain the detector. We thank the funding agencies for financial support of this experiment. We wish to thank the DESY directorate for the support and hospitality extended to the non-DESY members of the collaboration.

References

[1] I. Abt *et al.* [H1 Collaboration], Nucl. Phys. B **407** (1993) 515.
M. Derrick *et al.* [ZEUS Collaboration], Phys. Lett. B **316** (1993) 412.

[2] Y. L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641 [Zh. Eksp. Teor. Fiz. **73** (1977) 1216];
V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15** (1972) 1218 [Sov. J. Nucl. Phys. **15** (1972) 675];
V. N. Gribov and L. N. Lipatov, Yad. Fiz. **15** (1972) 781 [Sov. J. Nucl. Phys. **15** (1972) 438];
G. Altarelli and G. Parisi, Nucl. Phys. B **126** (1977) 298.

[3] A. De Rujula, S. L. Glashow, H. D. Politzer, S. B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D **10** (1974) 1649;
R. D. Ball and S. Forte, Phys. Lett. B **335** (1994) 77 [hep-ph/9405320].

[4] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44** (1976) 443;
E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45** (1977) 199;
Y. Y. Balitsky and L. N. Lipatov, Sov. Journ. Nucl. Phys. **28** (1978) 822.

[5] L. V. Gribov, E. M. Levin and M. G. Ryskin, Nucl. Phys. B **188** (1981) 555;
L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100** (1983) 1.

[6] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **21** (2001) 33 [hep-ex/0012053].

[7] H. Navelet, R. Peschanski and S. Wallon, Mod. Phys. Lett. A **9** (1994) 3393 [hep-ph/9402352].

[8] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C **13** (2000) 609 [hep-ex/9908059].

[9] S. Aid *et al.* [H1 Collaboration], Nucl. Phys. B **470** (1996) 3 [hep-ex/9603004].
C. Adloff *et al.* [H1 Collaboration], Nucl. Phys. B **497** (1997) 3 [hep-ex/9703012].

[10] J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. C **7** (1999) 609 [hep-ex/9809005].

[11] A. Donnachie and P. V. Landshoff, Z. Phys. C **61** (1994) 139 [hep-ph/9305319].

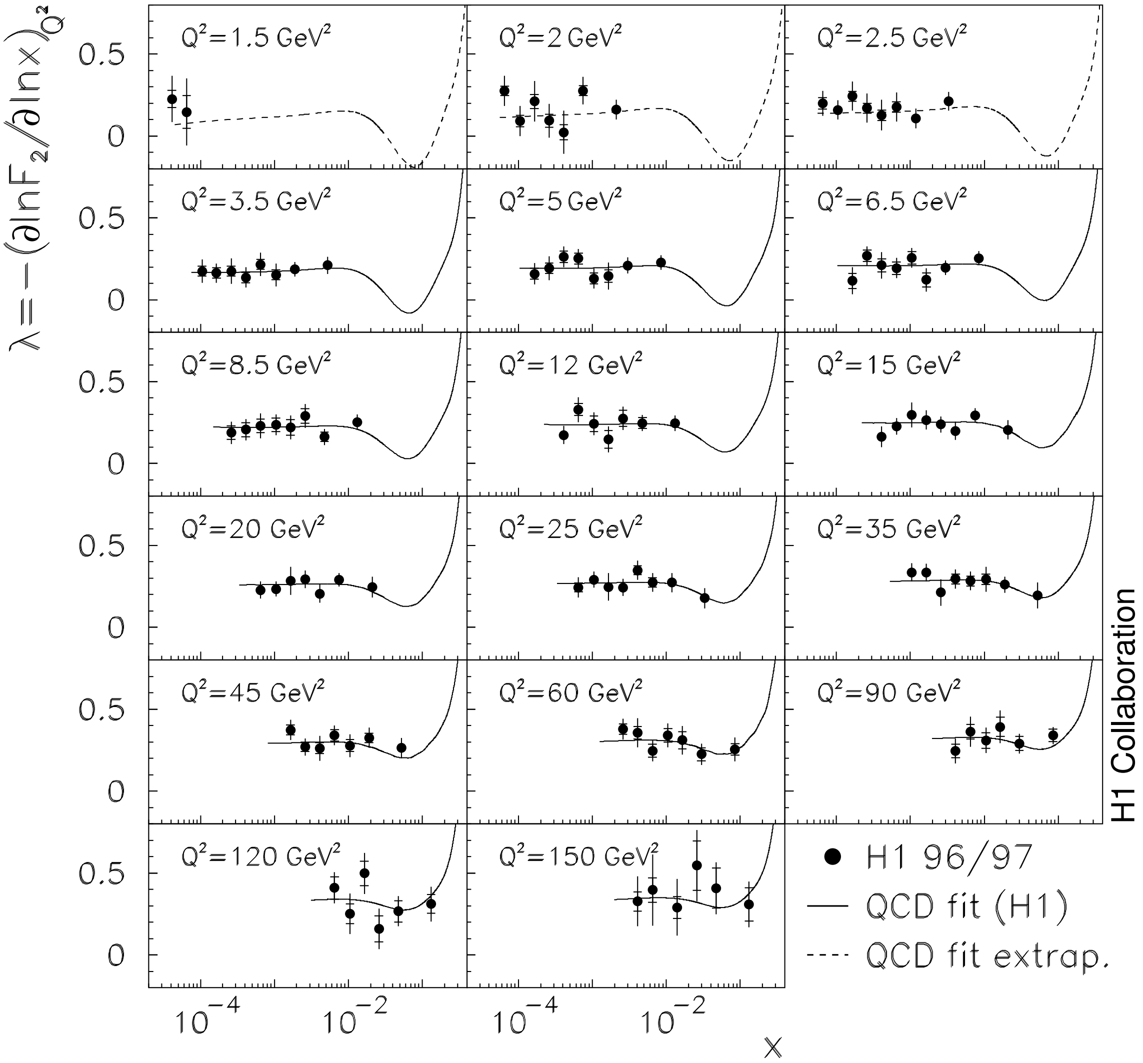

Figure 1: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the dashed curves represent the extrapolation of the QCD fit below $Q^2 = 3.5$ GeV^2.

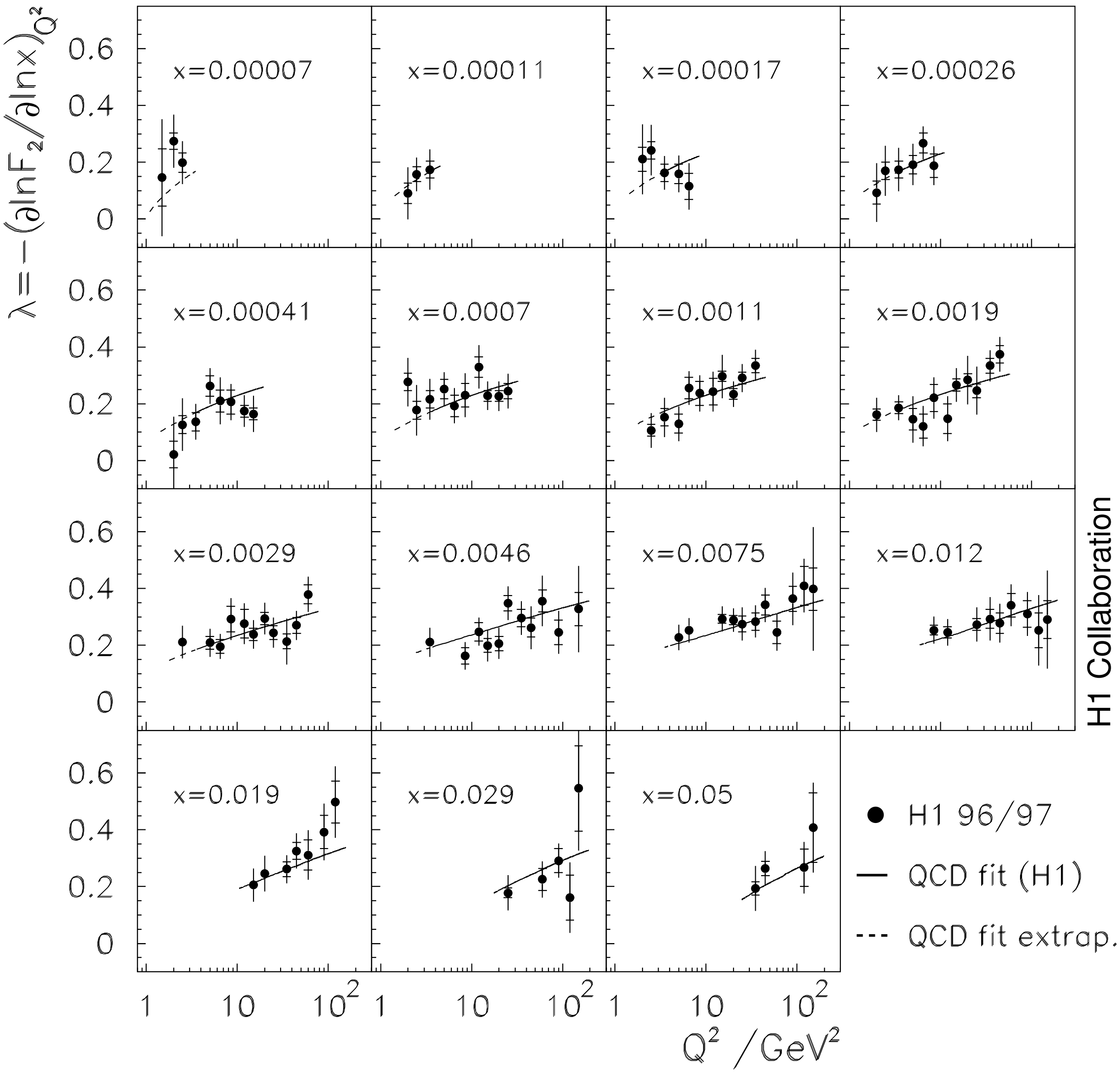

Figure 2: Measurement of the function $\lambda(x, Q^2)$: the inner error bars represent the statistical uncertainty; the full error bars include the systematic uncertainty added in quadrature; the solid curves represent the NLO QCD fit to the H1 cross section data described in [6]; the minimum Q^2 value of the data included in this fit is $Q^2 = 3.5$ GeV^2.

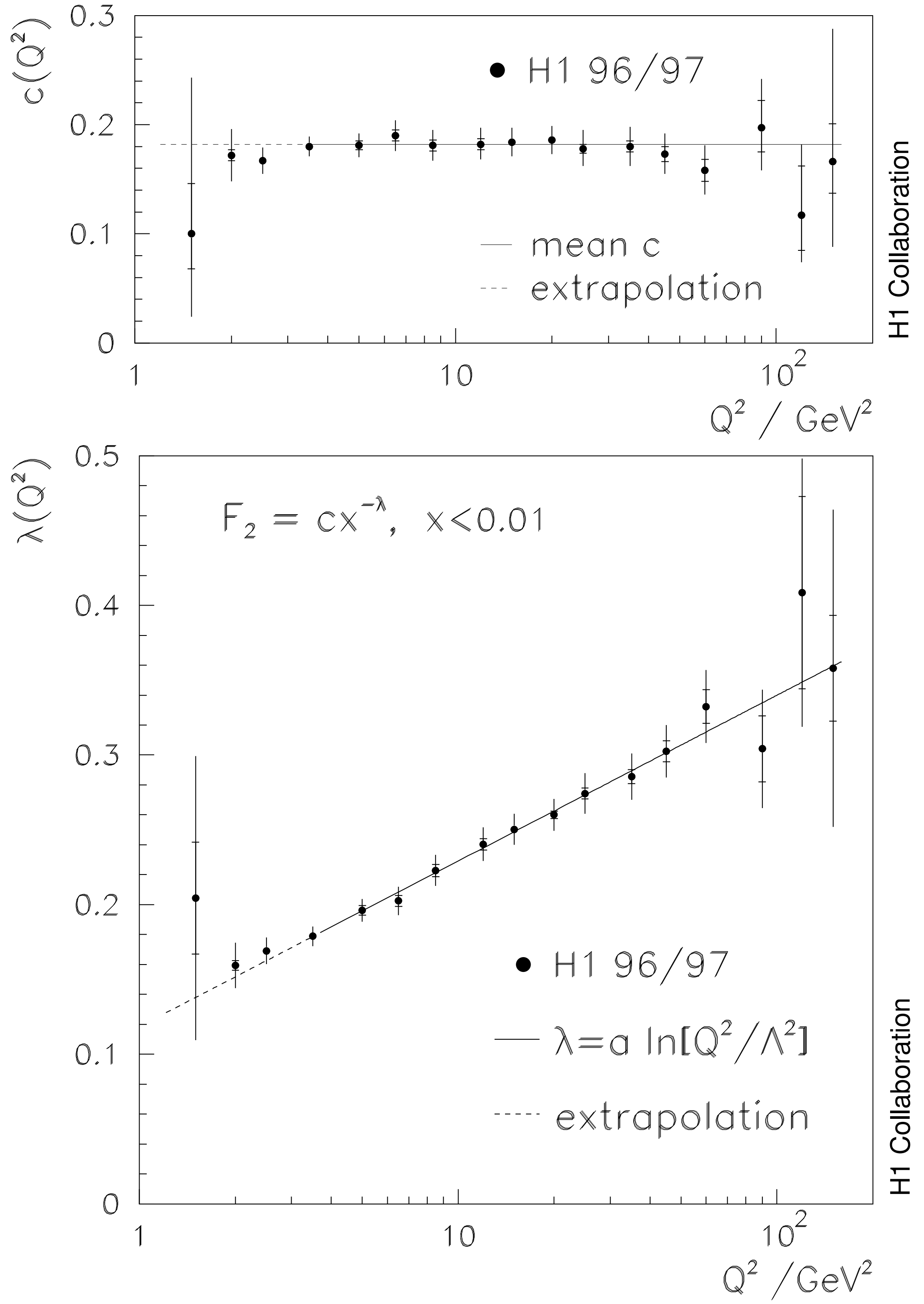

Figure 3: Determination of the coefficients $c(Q^2)$ (upper plot) and of the exponents $\lambda(Q^2)$ (lower plot) from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ to the H1 structure function data [6] for $x \leq 0.01$; the inner error bars illustrate the statistical uncertainties, the full error bars represent the statistical and systematic uncertainties added in quadrature. The straight lines represent the mean coefficient c (upper plot) and a fit of the form $a \ln[Q^2/\Lambda^2]$ (lower plot), respectively, using data for $Q^2 \geq 3.5$ GeV^2.

Q^2 [GeV^2]	x	λ	δ_{sta}	δ_{sys}	δ_{tot}
1.5	0.000041	0.225	0.052	0.131	0.141
1.5	0.000065	0.146	0.101	0.180	0.206
2.0	0.000065	0.274	0.029	0.089	0.093
2.0	0.000105	0.090	0.036	0.085	0.092
2.0	0.000165	0.211	0.043	0.116	0.124
2.0	0.000260	0.093	0.040	0.096	0.104
2.0	0.000410	0.022	0.047	0.124	0.132
2.0	0.000750	0.276	0.030	0.080	0.086
2.0	0.00210	0.161	0.020	0.057	0.060
2.5	0.000065	0.199	0.034	0.068	0.076
2.5	0.000105	0.158	0.026	0.053	0.059
2.5	0.000165	0.242	0.031	0.084	0.089
2.5	0.000260	0.170	0.030	0.082	0.088
2.5	0.000410	0.126	0.032	0.088	0.093
2.5	0.000650	0.177	0.032	0.084	0.090
2.5	0.00119	0.106	0.021	0.058	0.061
2.5	0.00329	0.211	0.013	0.057	0.058
3.5	0.000105	0.174	0.030	0.065	0.071
3.5	0.000165	0.163	0.031	0.050	0.059
3.5	0.000260	0.174	0.030	0.071	0.077
3.5	0.000410	0.136	0.032	0.054	0.063
3.5	0.000650	0.216	0.031	0.065	0.072
3.5	0.00105	0.152	0.031	0.063	0.070
3.5	0.00190	0.186	0.021	0.039	0.044
3.5	0.00525	0.210	0.012	0.050	0.051
5.0	0.000165	0.159	0.035	0.055	0.065
5.0	0.000260	0.192	0.033	0.065	0.073
5.0	0.000410	0.262	0.036	0.052	0.063
5.0	0.000650	0.251	0.034	0.049	0.060
5.0	0.00105	0.129	0.033	0.051	0.061
5.0	0.00165	0.145	0.038	0.072	0.081
5.0	0.00299	0.209	0.022	0.044	0.049
5.0	0.00849	0.227	0.013	0.042	0.044
6.5	0.000165	0.115	0.046	0.068	0.082
6.5	0.000260	0.268	0.036	0.047	0.059
6.5	0.000410	0.210	0.039	0.073	0.082
6.5	0.000650	0.193	0.038	0.049	0.062
6.5	0.00105	0.255	0.037	0.047	0.060
6.5	0.00165	0.121	0.042	0.058	0.072
6.5	0.00299	0.195	0.024	0.037	0.044
6.5	0.00849	0.253	0.014	0.040	0.043
8.5	0.000260	0.188	0.042	0.054	0.068
8.5	0.000410	0.206	0.042	0.048	0.064
8.5	0.000650	0.230	0.042	0.064	0.077
8.5	0.00105	0.237	0.042	0.049	0.064
8.5	0.00165	0.220	0.047	0.054	0.072
8.5	0.00260	0.292	0.045	0.058	0.073
8.5	0.00475	0.162	0.028	0.040	0.049
8.5	0.0132	0.253	0.017	0.045	0.048
12.0	0.000410	0.174	0.020	0.052	0.056
12.0	0.000650	0.330	0.036	0.067	0.076
12.0	0.00105	0.243	0.047	0.052	0.070
12.0	0.00165	0.147	0.053	0.059	0.079
12.0	0.00260	0.276	0.050	0.053	0.073
12.0	0.00475	0.247	0.032	0.038	0.049
12.0	0.0132	0.245	0.019	0.043	0.047
15.0	0.000410	0.164	0.020	0.060	0.064
15.0	0.000650	0.227	0.018	0.047	0.050
15.0	0.00105	0.296	0.018	0.074	0.076
15.0	0.00165	0.266	0.021	0.055	0.059
15.0	0.00260	0.238	0.021	0.045	0.050
15.0	0.00410	0.199	0.024	0.050	0.056
15.0	0.00750	0.292	0.016	0.041	0.044
15.0	0.0210	0.206	0.013	0.057	0.059
20.0	0.000650	0.227	0.019	0.048	0.052
20.0	0.00105	0.234	0.019	0.041	0.045
20.0	0.00165	0.284	0.022	0.082	0.085
20.0	0.00260	0.293	0.022	0.051	0.055
20.0	0.00410	0.206	0.025	0.049	0.055
20.0	0.00750	0.289	0.016	0.038	0.041
20.0	0.0210	0.246	0.012	0.063	0.064
25.0	0.000650	0.244	0.027	0.055	0.061
25.0	0.00105	0.291	0.020	0.045	0.050
25.0	0.00165	0.246	0.024	0.081	0.084
25.0	0.00260	0.244	0.024	0.047	0.053
25.0	0.00410	0.348	0.027	0.053	0.059
25.0	0.00650	0.274	0.028	0.051	0.058
25.0	0.0119	0.273	0.020	0.056	0.060
25.0	0.0329	0.178	0.016	0.060	0.062
35.0	0.00105	0.335	0.025	0.049	0.055
35.0	0.00165	0.334	0.026	0.047	0.054
35.0	0.00260	0.213	0.026	0.077	0.081
35.0	0.00410	0.295	0.030	0.051	0.059
35.0	0.00650	0.283	0.030	0.048	0.057
35.0	0.0105	0.293	0.033	0.070	0.077
35.0	0.0190	0.261	0.026	0.042	0.049
35.0	0.0525	0.194	0.023	0.076	0.079
45.0	0.00165	0.374	0.031	0.053	0.061
45.0	0.00260	0.270	0.028	0.043	0.052
45.0	0.00410	0.262	0.033	0.068	0.075
45.0	0.00650	0.342	0.035	0.049	0.060
45.0	0.01050	0.277	0.037	0.058	0.069
45.0	0.0190	0.325	0.029	0.055	0.062
45.0	0.0525	0.263	0.025	0.056	0.061
60.0	0.00260	0.379	0.033	0.054	0.063
60.0	0.00410	0.356	0.038	0.081	0.089
60.0	0.00650	0.246	0.040	0.051	0.064
60.0	0.0105	0.340	0.042	0.061	0.074
60.0	0.0165	0.311	0.053	0.070	0.088
60.0	0.0299	0.225	0.039	0.052	0.065
60.0	0.0849	0.255	0.035	0.064	0.073
90.0	0.00410	0.245	0.043	0.063	0.076
90.0	0.00650	0.363	0.044	0.081	0.092
90.0	0.0105	0.310	0.047	0.061	0.077
90.0	0.0165	0.392	0.058	0.081	0.100
90.0	0.0299	0.291	0.042	0.043	0.060
90.0	0.0849	0.340	0.039	0.035	0.052
120.0	0.00650	0.410	0.068	0.066	0.095
120.0	0.0105	0.252	0.061	0.107	0.124
120.0	0.0165	0.498	0.074	0.102	0.126
120.0	0.0260	0.161	0.079	0.095	0.124
120.0	0.0475	0.267	0.065	0.063	0.091
120.0	0.132	0.312	0.056	0.091	0.106
150.0	0.00410	0.327	0.059	0.141	0.153
150.0	0.00650	0.397	0.075	0.204	0.218
150.0	0.01400	0.291	0.067	0.159	0.173
150.0	0.0260	0.545	0.150	0.158	0.218
150.0	0.0475	0.408	0.122	0.102	0.159
150.0	0.132	0.308	0.101	0.096	0.139

Table 1: Measurement of the derivative $\lambda = -(\partial \ln F_2/\partial \ln x)_{Q^2}$ at fixed Q^2. For the systematic uncertainties the correlations between adjacent x values are taken into account. The total error is the squared sum of the statistical and systematic uncertainties, given as absolute values.

$Q^2[GeV^2]$	c	δ^c_{sta}	δ^c_{tot}	λ	δ^λ_{sta}	δ^λ_{tot}
1.5	0.10	+ 0.05 − 0.03	+ 0.14 − 0.08	0.20	0.04	0.10
2.0	0.172	0.005	0.024	0.159	0.003	0.015
2.5	0.167	0.003	0.012	0.169	0.002	0.009
3.5	0.180	0.003	0.009	0.179	0.002	0.007
5.0	0.181	0.004	0.011	0.196	0.003	0.008
6.5	0.190	0.005	0.014	0.202	0.004	0.009
8.5	0.181	0.005	0.014	0.223	0.004	0.010
12.0	0.182	0.005	0.015	0.240	0.004	0.011
15.0	0.184	0.003	0.013	0.250	0.002	0.010
20.0	0.186	0.003	0.013	0.260	0.003	0.011
25.0	0.178	0.004	0.017	0.274	0.004	0.014
35.0	0.180	0.005	0.018	0.286	0.005	0.016
45.0	0.173	0.007	0.019	0.302	0.007	0.017
60.0	0.158	0.010	0.023	0.332	0.011	0.024
90.0	0.197	+ 0.025 − 0.022	+ 0.045 − 0.039	0.304	0.022	0.040
120.0	0.117	+ 0.045 − 0.032	+ 0.065 − 0.043	0.408	0.064	0.089
150.0	0.17	+ 0.04 − 0.03	+ 0.12 − 0.08	0.36	0.04	0.11

Table 2: The coefficients c and exponents λ from fits of the form $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$ using H1 F_2 data [6], for $x \leq 0.01$, taking into account the systematic error correlations. Here δ_{sta} denotes the statistical uncertainty and δ_{tot} comprises all uncertainties added in quadrature. The uncertainties are given as absolute values. They are symmetric to very good approximation, apart from the uncertainties of the coefficient $c(Q^2)$ at the edges of the Q^2 region.