**Diss. ETH No. 13668 and**
**ETHZ-IPP Internal Report 2000-04**

# Muon Pair Production with high Invariant Mass in $e^+p$ – Collisions at HERA

A dissertation submitted to the

**Swiss Federal Institute of Technology Zurich**

for the degree of

**Doctor of Natural Sciences**

presented by

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April 2000

**Abstract**

Many processes in physics beyond the standard model show the signature of isolated leptons. A possible excess of events with one isolated lepton and high missing transverse energy was reported in 1998 by the H1 collaboration.
Possible explanations are statistical fluctuations, detector effects or new physics beyond the standard model. The last two explanations could also be visible in the channel with two isolated leptons.

This analysis measures the cross section for the production of two isolated muons in the elastic and inelastic channel in the kinematic range $20^\circ < \theta^\mu < 160^\circ$, $M^{\mu\mu} > 5$ GeV/c$^2$ and $p_T^\mu > 1.8$ GeV/c. The data were collected during the years 1996 and 1997, which corresponds to an integrated luminosity of $32.4 \pm 0.9$ pb$^{-1}$. The result is

$$\begin{aligned} \sigma_{vis}^{el}(e^+p \to \mu^+\mu^-X) &= (26.6 \pm 2.2 \ \pm 2.9)\ pb \\ \sigma_{vis}^{in}(e^+p \to \mu^+\mu^-X) &= (20.2 \pm 2.4 \ \pm 3.6)\ pb \end{aligned}$$

which is in good agreement with the expectation from the standard model. Also the differential distributions with respect to $P_T^X$, $P_T^{miss}$ and $M^{\mu\mu}$ agree with the expectation.

No signal for new physics has been found.

**Zusammenfassung**

Viele Prozesse in der Physik ausserhalb des Standardmodells zeigen eine Signatur mit isolierten Leptonen. Ein möglicher Überschuss von Ereignissen mit einem isolierten Lepton und hohem fehlenden Transversalimpuls wurde 1998 durch die H1 Kollaboration publiziert.
Mögliche Erklärungen sind statistische Fluktuationen, Detektoreffekte oder neue Physik ausserhalb des Standardmodells. Die zwei letzteren könnten auch im Kanal mit zwei isolierten Leptonen sichtbar sein.

Diese Analyse misst den Wirkungsquerschnitt für die Produktion von zwei isolierten Muonen im elastischen und inelastischen Kanal im kinematischen Bereich von $20^\circ < \theta^\mu < 160^\circ$, $M^{\mu\mu} > 5$ GeV/c$^2$ und $p_T^\mu > 1.8$ GeV/c. Die Daten wurden in den Jahren 1996 und 1997 aufgezeichnet, was einer integrierten Luminosität von $32.4 \pm 0.9$ pb$^{-1}$ entspricht. Das Resultat ist

$$\begin{aligned} \sigma_{vis}^{el}(e^+p \to \mu^+\mu^-X) &= (26.6 \pm 2.2 \ \pm 2.9)\ pb \\ \sigma_{vis}^{in}(e^+p \to \mu^+\mu^-X) &= (20.2 \pm 2.4 \ \pm 3.6)\ pb \end{aligned}$$

was in guter Übereinstimmung mit der Erwartung aus dem Standardmodell ist. Auch die differentiellen Verteilungen bezüglich $P_T^X$, $P_T^{miss}$ und $M^{\mu\mu}$ stimmen mit den Erwartungen überein.

Kein Anzeichen für neue Physik wurde gefunden.

# Contents

# Chapter 1

# Motivation

## 1.1 The Search for Physics beyond the Standard Model

The concept of *local gauge symmetry* has proven as one of the most fruitful concepts in modern particle physics. The so called *Standard Model (SM)* of particle physics consists of a local $SU(2)_L \otimes U(1)_Y$ symmetry for electroweak interaction (broken down to $U(1)_{em}$ electromagnetic interaction) and a $SU(3)_C$ gauge symmetry for strong interaction. Apart from some recent results in neutrino physics, no deviation from the *SM* has been observed until now.
However, from a theoretical point of view, the *SM* can hardly be the end of physics. A "*good*" theory should explain the huge number of free parameters in the *SM* (at least 19), especially the particle masses (which one would expected to be at the Planck scale), the origin of the six flavours of quarks and leptons and their weak charged current mixing and CP violation. Also it is tempting to try to unify all the particle interactions in a simple gauge group.
For this reason, theoretical physicists are looking for theories beyond the standard model, from which the later appears naturally as a low energy limit. The most popular extensions are the Supersymmetric Theories. Many of these theories predict new phenomenology at an energy scale close to the center of mass energy of todays particle colliders. Therefore, one is looking with great effort for new resonances or new event signatures. So far, searches for supersymmetric particles have led only to exclusion limits in parameter spaces. However, there are observations of very few events, which are not fully understood. It is the measurement of an event with the signature $p\bar{p} \rightarrow \gamma\gamma ee+$ high missing transverse momentum at CDF [3] and the observation of a possible excess of events with an isolated lepton and missing transverse momentum at the H1 detector [4]. These events could give a hint on new physics, although they have not much predictive power, because of the very poor statistic.
The significance of these measurements strongly depends on the knowledge of the background. Looking at the H1 measurement, one of the main background sources

is the QED production of muon pairs where one muon escapes the detector unseen. In the next section we will focus a little bit closer on the isolated lepton measurement. We will see how this motivates the study of lepton pair production with high invariant mass, which is done in this analysis.

## 1.2 Observation of Events with an Isolated High Energy Lepton and Missing Transverse Momentum at HERA

As reported in [4] a search for events with an isolated high energy lepton and an imbalance in transverse momentum has been performed with the H1 detector at HERA. A very short overview is given here.

The data was collected between 1994 and 1997, corresponding to an integrated luminosity of $41 \pm 1$ pb$^{-1}$. In this thesis the *ep* running period from 1999 has been added which corresponds to an integrated luminosity of another $16 \pm 1$ pb$^{-1}$. The selection criteria are

- $p_T^{calo} > 25$ GeV/c (20 GeV/c for muon production in 1999). $p_T^{calo}$ is the vector sum of the energy deposited in the central calorimeters.
- There is at least 1 track with $p_T > 10$ GeV/c and $\theta > 10°$, where $p_T$ is the transverse momentum as measured by the tracker and $\theta$ is the polar angle.
- A reconstructed vertex has to be found in the *ep* interaction region, and a set of topological and timing requirements are applied, in order to reject non-*ep* events.
- to reject NC events with energy lost in calorimeter cracks or in the beam pipe, events are rejected if there is an identified scattered positron and one of the following conditions is fulfilled:
  - the momentum from the positron is balanced by the hadronic system in azimuth within $\Delta\Phi = 5°$. The scattered quark and the scattered positron are moving back to back.
  - $\delta := \sum_i E_i(1 - \cos\theta_i) > 45$ GeV/c, where $E_i$ and $\theta_i$ denote the energy and polar angle of the particles in the final state. For events with the scattered positron in the detector, $\delta$ peaks at twice the initial positron energy, that is, at 54 GeV/c.

In the remaining sample the isolation of the high $p_t$ tracks was studied. The used observables were

- $D_{jet}$, their distance to the closest hadron jet in the pseudorapidity - azimuth plane: $D_{jet} = \sqrt{\Delta\eta^2 + \Delta\phi^2}$
- $D_{track}$, their distance in $\eta - \phi$ to the closest track

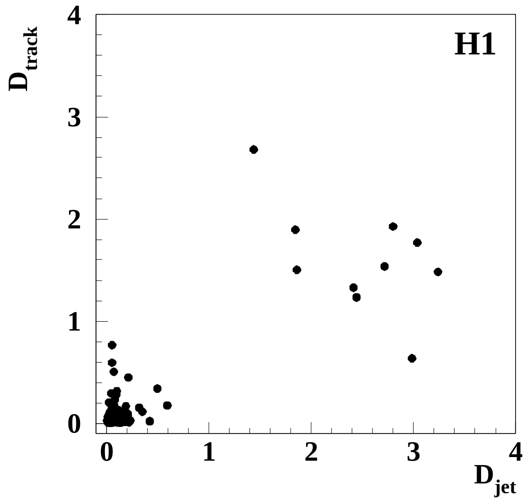


Figure 1.1: Distance to the closest hadronic jet ($D_{jet}$) and to the closest track ($D_{track}$) for all high $p_T$ tracks in the sample

The bulk of events with an intrinsic imbalance of transverse momentum comes from charged current (CC) interactions. There, an isolated muon can only originate from heavy quark or $\pi/K$ decays. Thus, one would expect, that the high-$p_T$ tracks are not clearly isolated. Figure 1.1 shows the measurement. Indeed, in most cases the tracks are not isolated. But 10 events have the signature of **one** high-$p_T$ track which is well separated from all other activities in the detector.
In 3 events this track is assigned to an electron candidate, while the other 7 events are assigned to an isolated muon. The production of this type of events has been studied within the standard model. In table 1.1 the observed and predicted rates are shown for the electron and the muon channel. The dominating contributions are W production ($ep \to el\nu_l X$, $ep \to \nu_e l\nu_l X$) and QED lepton pair production ($ep \to el^+l^-X$) where one of the two leptons escapes the detector undetected.
Other contributions come from neutral current ($ep \to eX$) and charged current ($ep \to \nu_e X$) interactions, where the lepton is a misidentified hadron, heavy quark production with subsequent semileptonic decays, Z production ($ep \to eZX$, $Z \to l^+l^-$) where one lepton escapes the detector unseen and photo production of two

| | muons | electrons |
|---|---|---|
| Data | 7 | 3 |
| W Production | $0.83 \pm 0.17$ | $2.34 \pm 0.67$ |
| Z Production | $0.01 \pm 0.01$ | $0.01 \pm 0.01$ |
| Charged Current interaction (CC) | $0.01 \pm 0.01$ | $0.03 \pm 0.02$ |
| Neutral current interaction (NC) | $0.18 \pm 0.12$ | $0.75 \pm 0.14$ |
| Photoproduction | $< 0.03$ | $< 0.03$ |
| Heavy Quarks | $< 0.06$ | $< 0.05$ |
| Lepton Pair Production in QED | $0.28 \pm 0.07$ [1] | $0.19 \pm 0.06$ |
| all MC | 1.40±0.22 | $3.40 \pm 0.69$ |

Table 1.1: Observed and predicted event rates. The numbers refer to the summed production of positive and negative leptons. The limits correspond to 95% confidence level. In the muon channel the main sources are W production and lepton pair production. The asymmetry between electron and muon channel is due to the cut on $p_T^{calo}$. In the muon channel, it acts more or less as a $p_T^X$ cut.

jets, where one fluctuates into a single hadron which is misidentified as a lepton.

The observed rate in the muon channel is roughly a factor of 5 higher than what is expected from Monte Carlo simulations. The probability to see at least 7 muons with an expectation of 1.40±0.22 events is calculated as

$$\mathcal{P}(N \geq 7) = \int_0^\infty dx G(x; 1.40, 0.22) \sum_{k=7}^{\infty} p(k; x),$$

where

- $G(x; 1.40, 0.22)$ is the probability density function for the expectation value. It is a Gaussian with mean 1.40 and width 0.22.
- p(k;x) is the Poisson probability to observe $k$ events with an expectation of $x$ events.

The result is $\mathcal{P}(N \geq 7) = 8.4 \cdot 10^{-4}$.
Figures 1.2 and 1.3 show the distributions of the hadronic transverse momentum $P_T^X$, (which is the transverse part of the vector sum of the energy deposited in the LAr and tail catcher calorimeter not assigned to a lepton) and the missing transverse momentum $p_T^{miss}$ in the muon channel. There is not only a mismatch in the normalization, but also a tendency to higher values of $p_T^X$ in the data.
Figures 1.4 and 1.5 show the same distributions in the electron channel. The normalizations are in good agreement, while the shape again tends to higher $p_T^X$ in the data.

[1] The error as stated in the H1 paper is substantially higher. Due to low statistics for tests of the LPAIR generator, the inelastic contribution was conservatively allowed to vary within a factor of 2. Due to the present analysis, this error could be lowered.

Possible explanations for an excess of events could be:

1. It is just a statistical fluctuation. As stated before the probability for this to occur is very small.
2. The long tail in the $P_T^X$ distribution in the data could be a detector effect.
3. There are models beyond the standard model, which produce events with the signature of isolated muons:
   - According to [5], three of the muon events are compatible with the hypothesis of production of excited quarks ($q^* \to qW \to q\mu\nu_\mu$) within compositeness models.
   - In Rp violating SUSY, this signature may occur in the decay of a squark $\tilde{u}_L \to d\chi_1^+$ and the subsequent gauge decay $\chi_1^+ \to W^+\chi_1^0 \to l^+\nu\chi_1^0$ (see decay channel S6 in [6]).

From this, the following questions arise:

1. How good is the measurement of muons in H1? Is the tail at high $P_T^X$ a detector effect? Misidentified leptons could add a substantial imbalance in hadronic energies.
2. Is the description of the background in the Monte Carlo simulation correct? Of special interest here is the distribution of $p_T^X$ and $p_T^{miss}$.
3. If $W$'s are produced from excited quarks, there should also be $Z^0$ production. Is this seen?
4. In the Rp-violating scenario, the down type quark could be a $b$ quark decaying into $c\mu\nu$. This would add to the channel with two isolated muons and high missing energy. Are there any such events?

Muon pair production in EW theory is a dedicated process to answer the first two questions because

1. The cross section is well known. There are few theoretical uncertainties because of higher order corrections and the background is very low. This predestines it to check the measurement of muons.
2. If the tails in the $P_T^X$ distributions are detector effects, this would also be visible in the two muon channel. Thus, we have a possibility to investigate and eventually exclude this effect.

3. the Monte Carlo generator for this process is the least tested generator at these energies. It contains only two photon physics. The question has to be answered, weather other graphs contribute substantially at higher energies. Since QED is one of the main background sources, this could alter the distributions in figure 1.2 and 1.3. A previous analysis about muon pair production ([20]) leaves room for this, since a mismatch in the total cross section of about 60% has been found.

The following analysis investigates exactly these questions. In chapter 2 this is done on generator level. The influence of other processes is analyzed. In chapter 3 a description of the detector follows. The measurement of muons is investigated extensively in chapter 4. In chapter 5 the muon production cross section is determined. Finally, events with three muons have been found. Since multi muon events are very rare, some of the muons should be misidentified hadrons. This is analyzed in chapter 6.

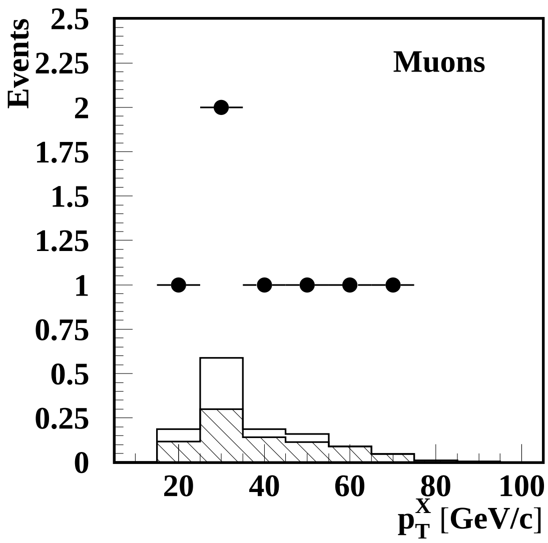


Figure 1.2: Differential distribution of $p_T^X$ for isolated muon production. Dots represent data, the dashed histogram shows W production and the open histogram shows NC and two photon events added to it. Aside from the normalization there is a tendency to higher values in the data.

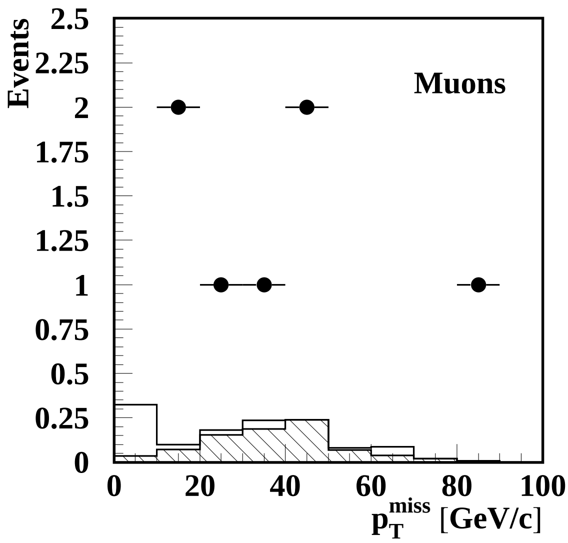


Figure 1.3: Differential distribution of $p_T^{miss}$ for isolated muon production. Dots represent data, the dashed histogram shows W production and the open histogram shows NC, two photon events and W production.

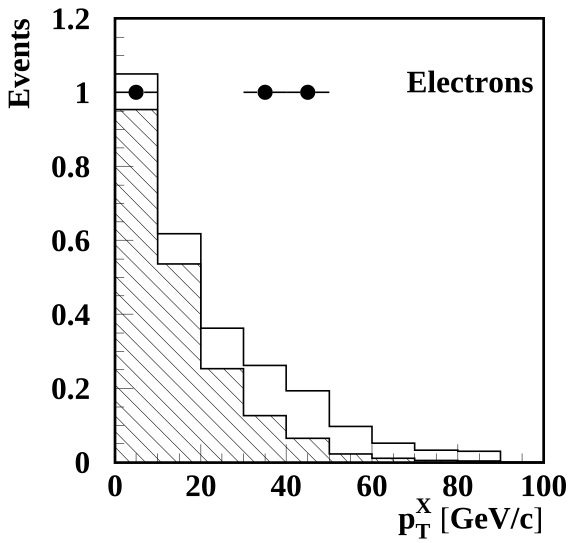


Figure 1.4: Differential distribution of $p_T^X$ for isolated electron production. Dots represent data, the dashed histogram shows W production and the open histogram shows NC and two photon events added to it. The integral above $p_T^X$=20 GeV/c in Monte Carlo is 0.29 events compared to 2 events in data.

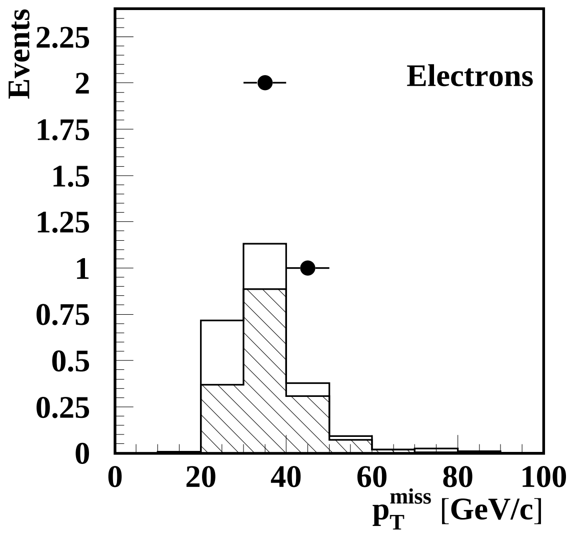


Figure 1.5: Differential distribution of $p_T^{miss}$ for isolated electron production. Dots represent data, the dashed histogram shows W production and the open histogram shows the sum of W production NC and two photon events.

# Chapter 2

# Muon - Pair Production in QED

## 2.1 Production Processes

This chapter describes the production of muon pairs on generator level. So far, in H1 the generator LPAIR has been used for this process. It is based on the work of J.A.M. Vermaseren [22] and includes only two photon physics (so called Bethe-Heitler process). This is the dominating contribution to muon pair production in QED.

In figure 2.1 other non-resonant processes are shown. The relative weight of these graphs have been investigated in [21] as a function of the transverse momentum $p_T$ of one of the two muons. Figure 2.2 is taken from there and shows the various contributions to $p_T^3 \frac{d\sigma}{dp_T}$ as a function of the transverse momentum $p_T$ of one of the two generated leptons.

The differential cross section for direct and resolved [1] Drell-Yan processes (figure 2.1 e,f) is about two orders of magnitude smaller than for the two photon graphs up to $p_T$=25 GeV/c. Then it rises but stays an order of magnitude smaller at $p_T = 40$ GeV/c. They are neglected in the following.
For $p_T < 20$ GeV/c the contribution of the Compton like processes (figure 2.1 c,d) is smaller than 1% but reaches the same order of magnitude at 40 GeV/c. In a previous analysis ([20]) the measurement of the muon pair production cross section was about 60% higher than expected. It was supposed that it could be the influence of these graphs. This question is answered in the following.

## 2.2 Monte Carlo

At the time of the beginning of this analysis the only available Monte Carlo generator for lepton pair production in ep colliders was LPAIR. This will be extended

[1] *direct* means the pointlike interaction with the photon, while *resolved* means the interaction with a quark within the photon structure

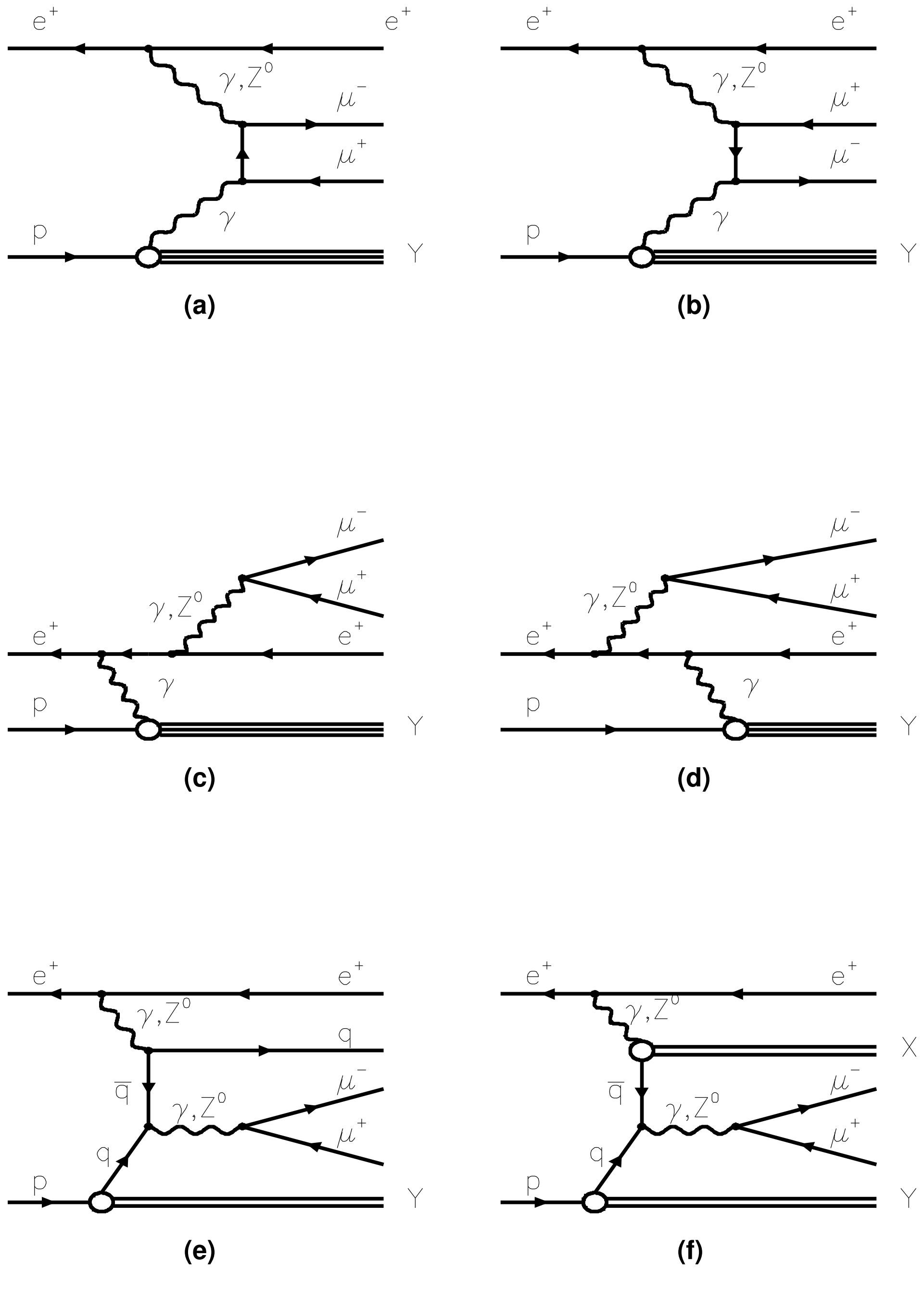


Figure 2.1: Feynman graphs for different production processes: Bethe-Heitler process (a) and (b), Compton like processes (c) and (d), Drell-Yan direct (e) and resolved (f).

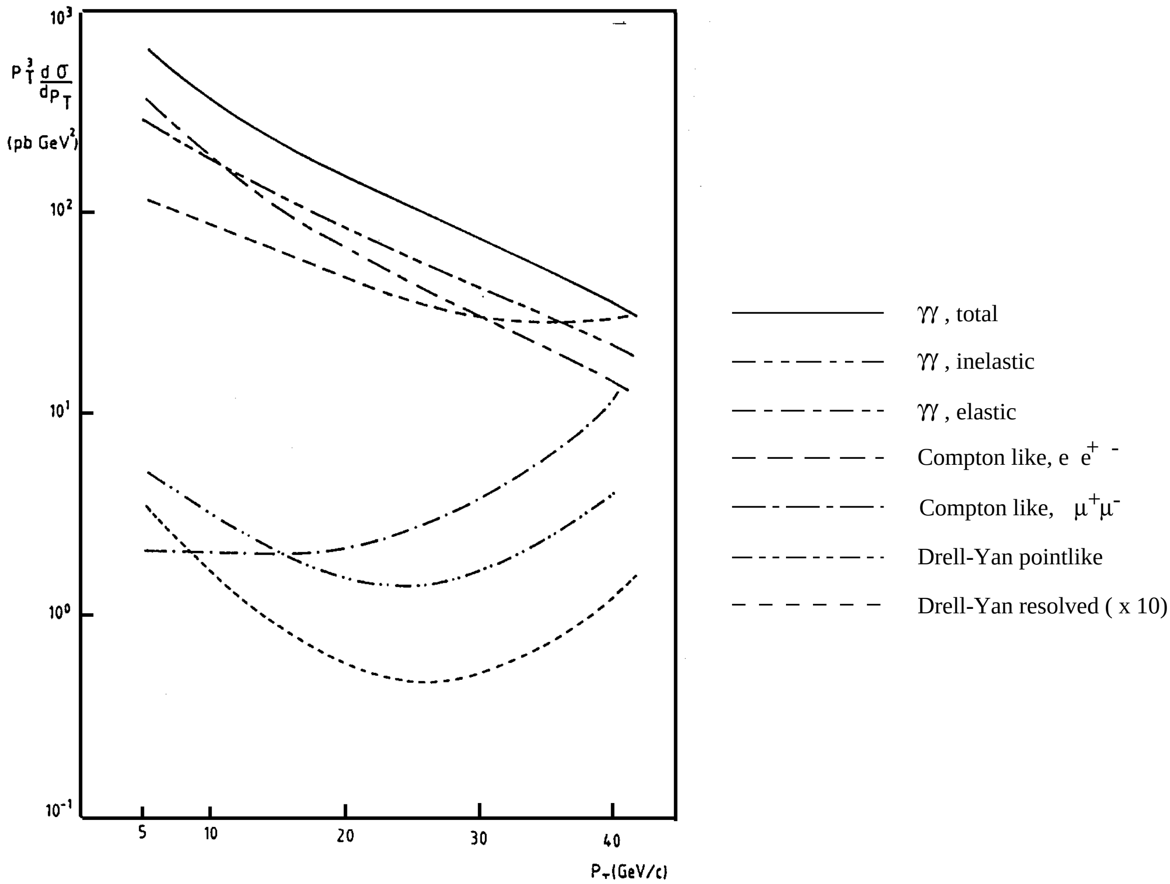


Figure 2.2: Various contributions to $p_T^3 \frac{d\sigma}{dp_T}$ for lepton pair production as a function of the transverse momentum of one of the two leptons.

here. In a first step the matrix element has been rewritten. In the following this generator is called LPAIR++. Since in the muon channel the Compton like graphs did not add a severe peaking behaviour of the matrix element no new phase space generator has been developed.
In the meantime an independent generator GRAPE ([31]) is available. The extended generator LPAIR++ has been cross checked with GRAPE. The result has been used to estimate a systematical error in the expectation and can be found in chapter 5.

Apart from Drell-Yan processes, the event structure for muon pair production looks like in figure 2.3. There are two systems in the final state. The system $X$ is the part which was altered in LPAIR++. A short description of how this is done is given in section 2.2.1.

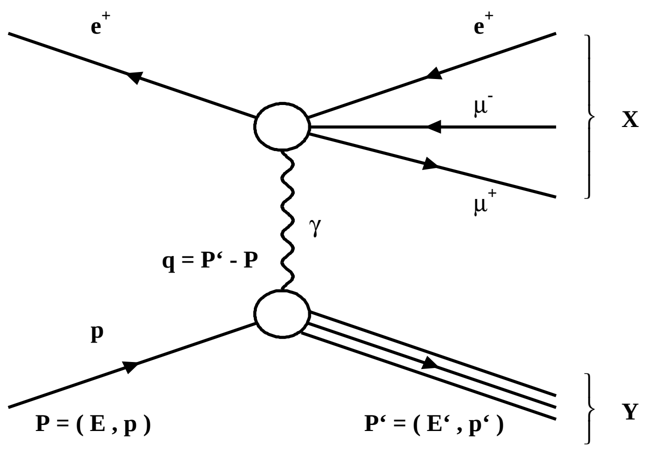


Figure 2.3: Schematics of an event in muon pair production.

The system $Y$ describes the final state of the scattered proton. In case of elastic scattering [2] the system Y consists only of a proton. This is dominant for low $Q^2$, which is defined as $Q^2 = -q^2$ and $q = P' - P$ is the four momentum transfered to the proton. For inelastic scattering two modes exist in LPAIR, which distinguish the resonance region at low $Q^2$ and mass $M_Y$ of the system $Y$ and the DIS (deep inelastic scattering) region at high $Q^2$. The description of this hadronic part is unaltered from LPAIR and is described in section 2.2.2.

### 2.2.1 Inclusion of Compton like Graphs

A new matrix element has to be written, which includes the graphs 2.1 (a) to (d) including the exchange of a $Z^0$. Although it is a leading order calculation it is non-trivial for two reasons.

1. The standard method, involving squaring the amplitude and summing over spin states, produces a large multiplicity of terms

2. there is very strong numerical cancelation between various terms. This can be so bad, that even double precision calculation would not suffice [22]

[2]Note, that *elastic* scattering here does not mean, that there is only a proton and an electron in the final state, but that the system $Y$ is a proton.

A method which addresses both items, is the *Helicity Method* [1]. This is briefly described in the following without going into much technical details. Also the proofs can be found in [1] and are not repeated here.

#### 2.2.1.1 Helicity Method

The trick is to compute $\mathcal{M}$ instead of $\sum |\mathcal{M}|^2$, where $\mathcal{M}$ is the matrix element, and the sum goes over external spins. This addresses point 2 above, since the cancelations will take place over less orders of magnitude.
It is not possible to express $\mathcal{M}$ in terms of scalar products of external momenta $p_i \cdot p_j$. Instead, we need something which is proportional to the square root of it. These are the spinor products $\bar{u}(p_j)u(p_j)$ which we are going to develop in the next subsections. From a computational point of view, we must find a representation where they are not much more complicated to evaluate than scalar products.

**Massless Spinors**
Because we will reduce the case of massive fermions to the case of massless fermions, we will start with the later.

For a spinor with lightlike momentum $p$ and helicity $\lambda$ we have

$$u_\lambda(p)\bar{u}_\lambda(p) = P_\lambda \not{p} \tag{2.1}$$

where $P_\lambda = \frac{1}{2}(1+\lambda\gamma^5)$ is the projection operator onto states with helicity $\lambda$.
Now, we choose two four-vectors $k_0^\mu$ and $k_1^\mu$ with the properties

$$k_0 \cdot k_0 = 0, \qquad k_1 \cdot k_1 = -1, \qquad k_0 \cdot k_1 = 0. \tag{2.2}$$

Then we define (up to its complex phase) a basic spinor with negative helicity through

$$u_-(k_0)\bar{u}_-(k_0) = P_- \not{k}_0 \tag{2.3}$$

and a positive helicity state through

$$u_+(k_0) = \not{k}_1 u_-(k_0) \tag{2.4}$$

which satisfies (2.1) because of (2.2).
For any lightlike momentum $p$ we can define the corresponding spinors through

$$u_\lambda(p) = \frac{1}{\sqrt{2p\cdot k_0}} \not{p} u_{-\lambda}(k_0) \tag{2.5}$$

which again satisfy (2.1). The only restriction is, that the denominator has to be nonzero.
One can find expressions for the spinor products

$$s(p_i,p_j) \equiv \bar{u}_+(p_i)u_-(p_j) = -s(p_j,p_i) \tag{2.6}$$

$$t(p_i,p_j) \equiv \bar{u}_-(p_i)u_+(p_j) = s^*(p_j,p_i) \tag{2.7}$$

which are quite compact if we choose

$$\begin{aligned} k_0^\mu &= (1,1,0,0) && (2.8)\\ k_1^\mu &= (0,0,1,0) && (2.9)\\ p_i^\mu &= (p_i^0, p_i^x, p_i^y, p_i^z) && (2.10) \end{aligned}$$

namely

$$s(p_i, p_j) = (p_i^y + ip_i^z)\left(\frac{p_j^0 - p_j^x}{p_i^0 - p_i^x}\right)^{1/2} - (p_i \leftrightarrow p_j) \tag{2.11}$$

The second term is the same as the first with the two momenta interchanged. Due to this explicit anti-symmetrization, the calculation is stable even if the two momenta become collinear.

**Massive Spinors**

For a fermion with momentum $q$ and mass $m$, we need a spinorial object which leads to the correct spin sum which is

$$\sum_\lambda u(q,\lambda)\bar{u}(q,\lambda) = \not{q} \pm m \tag{2.12}$$

for fermions $(+)$ and antifermions $(-)$. Here, $\lambda = \pm$ is the sign of the projection of the spin onto some fixed vector $s^\mu$. With some lightlike momentum $k$, a good spinor for a fermion is

$$u(q,+) = \frac{1}{\sqrt{2q\cdot k}}\,(\not{q} + m)u_-(k), \tag{2.13}$$

i.e. we can find a unique spin vector $s^\mu$ such that the Dirac equations hold. Next, we choose two lightlike momenta $p_1$ and $p_2$ which build up $q$ and choose $p_2$ for $k$:

$$p_1^2 = p_2^2 = 0, \qquad p_1^\mu + p_2^\mu = q^\mu, \qquad p_2^\mu = k^\mu \tag{2.14}$$

For this choice, the spin vector gives

$$s^\mu = (p_1^\mu - p_2^\mu)/m \tag{2.15}$$

One gets the other spin state / spinors for anti-fermions by changing $u_-(k)$ into $u_+(k)$ / $+m$ into $-m$ in (2.13). The spinor products of massive fermions have the same structure as (2.11) but with the corresponding spacelike momenta.

**The Matrix Element**

The calculation of $\mathcal{M}$ also solves problem 1 above, because the complexity grows only linearly with the number of graphs. Moreover, the same computer code can be used for the different graphs with permutations of the external momenta. We do not give all the details but show the key point with the example of DIS, where we have lepton quark scattering. This follows closely ref. [2], where it is used in a $e^+e^- \to e^+e^-l^+l^-$ generator.

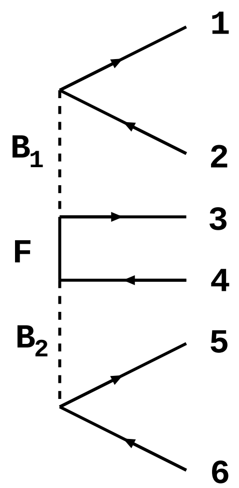


Figure 2.4: Generic graph for muon production in DIS.

Figure 2.4 shows the generic graph. Solid lines are fermions and dashed lines are bosons (photons or $Z^0$). All the feynman diagrams can be obtained by different assignments of the external momenta to the legs of the graph. For the propagators we can write

$$
\begin{aligned}
B_1 &= \frac{g^{\mu\nu}}{(b_1 p_1 + b_2 p_2)^2 - m_{B_1}^2 + i m_{B_1}\Gamma_1} \\
B_2 &= \frac{g^{\mu\nu}}{(b_5 p_5 + b_6 p_6)^2 - m_{B_2}^2 + i m_{B_2}\Gamma_2} \\
F &= \frac{Q}{(b_1 p_1 + b_2 p_2 + b_3 p_3)^2 - m_3^2}
\end{aligned}
$$

The factors $b_i$ are $+1$ $(-1)$ if $i$ is an outgoing (incoming) particle, $p_i$, $m_i$ are the momentum and mass assigned to leg $i$ of the graph, $m_{B_i}$, $\Gamma_i$ are the mass and width of the boson $i$ (0 for $\gamma$'s). $Q$ is the spinorial part of the lepton propagator. If the masses of anti-fermions are taken *negative*, it is

$$Q = b_1(\not{p}_1 + m_1) + b_2(\not{p}_2 + m_2) + b_3(\not{p}_3 + m_3)$$

If we write $u(q,s)$ for both, the fermionic and anti-fermionic spinors (the type shall be implied by the argument), the total spinorial part of $\mathcal{M}$ is

$$S = \bar{u}(p_1,s_1)\Gamma_1^\mu u(p_2,s_2) \times \bar{u}(p_3,s_3)\ \Gamma_{2\mu}\ Q\ \Gamma_{3\nu}\ u(p_4,s_4) \times \bar{u}(p_5,s_5)\Gamma_4^\nu u(p_6,s_6)$$

where $\Gamma_i$ are the vertex couplings:

$$
\begin{aligned}
\Gamma_i^\mu &= ie\gamma^\mu, \qquad \text{for photon couplings} \\
\Gamma_i^\mu &= \frac{-ig}{4\cos\theta_W}\gamma^\mu(1-\gamma^5), \qquad \text{for Z}^0 \text{ couplings}
\end{aligned}
$$

Using the relation (2.12) this can be written in the form

$$S = \sum_{\lambda} \sum_{i=1}^{3} b_i G(p_1, \lambda_1; \Gamma_1; p_2, \lambda_2; p_3, \lambda_3; \Gamma_2; p_i, \lambda) G(p_i, \lambda; \Gamma_3; p_4, \lambda_4; p_5, \lambda_5; \Gamma_6; p_6, \lambda_6)$$

with

$$G(p_1, \lambda_1; \Gamma_1; p_2, \lambda_2; p_3, \lambda_3; \Gamma_2; p_4, \lambda_4) = \bar{u}(p_1, \lambda_1)\Gamma_1^{\mu} u(p_2, \lambda_2)\bar{u}(p_3, \lambda_3)\Gamma_{2\mu} u(p_4, \lambda_4)$$

These functions can be expressed by spinor products as developed above. This is tabulated in [2] as a function of the spins.

#### 2.2.1.2 Comparison with 2-Photon Generator

The new Monte Carlo generator LPAIR++ has been compared to the 2 photon generator LPAIR. If the new graphs are disabled in LPAIR++ the two generators are in agreement within statistical errors in all modes.
The effect of the $Z^0$ exchange can be seen in figure 2.5. However, the $Z^0$ peak corresponds to a cross section of 2.3 fb. With the corrected integrated luminosity of 17.8 pb$^{-1}$ used in this work this corresponds to an expectation of 0.04 events and cannot be seen.

Figure 2.6 shows the same plot in the low invariant mass region. This plot has been made using the generator GRAPE which is available in the meantime and includes all electroweak graphs. A difference in the two generators is visible only in a mass range below $\approx 2$ GeV/c$^2$. This is due to the Compton like processes (figure 2.1 (c) and (d) with photons). Furthermore, it is only visible in DIS where the momentum transfered to the proton is relatively high.

### 2.2.2 The Representation of the Proton

Three different modes are available to describe the vertex at the proton side..

1. Elastic scattering
2. Inelastic scattering in the kinematic range $Q^2 < 5$ (GeV/c)$^2$
3. Deep inelastic scattering in the kinematic range $Q^2 > 5$ (GeV/c)$^2$

$Q^2$ is the negative of the momentum transfered to the proton. The modes are described in the following. For a more detailed description see [23], [24], [25], [26], [27].

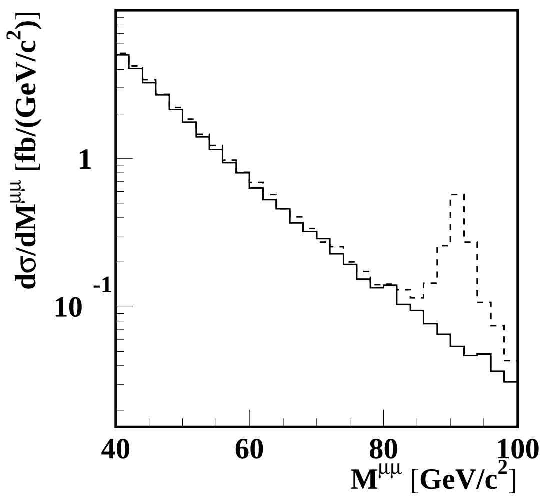


Figure 2.5: Cross section for $\mu$ pair production in the high invariant mass region in Monte Carlo simulation. The dashed histogram represents production with electroweak graphs included (LPAIR++), while the solid histogram represents the contribution from the two photon graphs only (LPAIR).

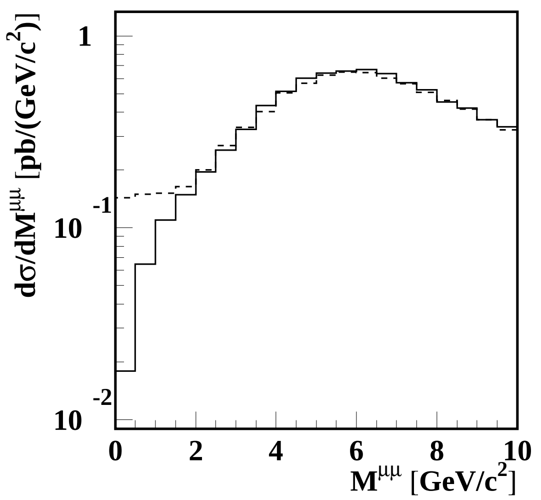


Figure 2.6: Cross section for $\mu$ pair production in the low invariant mass region in DIS Monte Carlo simulation. The dashed histogram represents production with Compton like graphs included (GRAPE), while the solid histogram represents the contribution from the two photon graphs only (LPAIR).

#### 2.2.2.1 Elastic Scattering

The parametrization chosen here is

$$\frac{d\sigma_{el}}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_p \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\Theta}{2}\right]$$

with

$\Theta$ the angle between the incoming positron and the outgoing leptonic system X (e$\mu\mu$) in the proton rest frame

$\left.\frac{d\sigma}{d\Omega}\right|_p$ the differential cross section of a pointlike fermion with the proton mass

$\tau = \frac{Q^2}{4m_P^2}$, where $m_P$ is the proton mass

$G_E$ and $G_M$ the electric and magnetic form form factors of the proton. They are approximated by the dipole factors

$$G_E(Q^2) = \frac{G_M(Q^2)}{|\mu_P|} \approx G_D(Q^2) = \frac{1}{(1 + Q^2/0.71(GeV/c)^2)^2}$$

here $\mu_P$ is the magnetic moment of the proton.

#### 2.2.2.2 Inelastic Scattering

The inelastic cross section can be written as

$$\frac{d^2\sigma_{in}}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \cdot \frac{E'}{E} \cdot \left(W_2(Q^2,\nu)\cos^2\frac{\Theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\Theta}{2}\right)$$

with

$\nu = E - E'$

$E$, $E'$ are the energies of the incoming positron and the outgoing leptonic system X (e$\mu\mu$) in the rest frame of the proton

The structure functions $W_1$ and $W_2$ are parametrized in [26] and fitted to experimental data.
The resonance region, defined through $Q^2 < 6$ (GeV/c)$^2$ and $m_Y < 2$ GeV/c$^2$, is parametrized separately in [27].

$$\sigma_{tot} = G_D^2 \exp\left[a(m_Y) + b(M_Y)\ln\frac{|\vec{q}|}{|\vec{q}_0|} + c(m_Y)\left(\left|\ln\frac{|\vec{q}|}{|\vec{q}_0|}\right|\right)^3\right]$$

with

$G_D$ the dipole form factor of the nucleon

$q = \begin{pmatrix} q_0 \\ \vec{q} \end{pmatrix}$ the 4-momentum transfered to the proton

Parameters $a, b, c$ witch depend only on the mass $m_Y$ and are tabulated in ref. [27].

#### 2.2.2.3 Deep Inelastic Scattering

The disadvantage of the inelastic mode is the fact that it doesn't supply a description of the hadronic final state. That's why another mode in the high $Q^2$ range was introduced, the DIS mode. In this mode the differential cross section for positron quark scattering is calculated and folded with the parton density function $f_{q/p}$ for the scattered quark according to

$$\frac{d\sigma_{ep}}{dQ^2} = \int \frac{d\sigma_{eq}}{dx dQ^2} \cdot f_{q/p}(x, Q^2) dx$$

The parton density function chosen here is CTEQ4 [28].
This description of the cross section is valid only in the high $Q^2$ regime. The cut value of 5 $(\mathrm{GeV/c})^2$ is justified by comparing the inelastic mode with the DIS mode as a function of $Q^2$. The cut was made in a region where they coincide (see figure 2.7).

### 2.2.3 Distribution of Generated Variables

At the end of this chapter some distributions are presented. The events have been generated with the new generator LPAIR++. The following cuts have been used:

1. invariant mass: $M^{\mu\mu} > 4$ GeV/c$^2$
2. polar angle: $10° < \theta^\mu < 170°$ for both muons
3. transverse momentum: $p_T^\mu > 1.0$ GeV/c for both muons
4. in the DIS mode: $Q^2 > 5$ $(\mathrm{GeV/c})^2$

In figure 2.7 we see the generated distribution of $log\left(\frac{Q^2}{GeV^2}\right)$. The open histogram shows events generated in the inelastic mode, the dashed histogram represents the

DIS mode. The absolute normalization is arbitrary but the relative normalization is correct. The cut has been made at $Q^2 = 5$ (GeV/c)$^2$ indicated by the solid line. In the following **'inelastic mode'** always means the inelastic mode for $Q^2 < 5$ (GeV/c)$^2$ and the DIS mode for $Q^2 > 5$ )(GeV/c)$^2$.
Figure 2.8 shows the distributions of the polar angles $\theta^{\mu}$, the transverse momentum $p_T^{\mu}$ and the invariant mass $M^{\mu\mu}$ of the generated muon pair and the opening angle $\theta^{\mu\mu}$ in three dimensions.

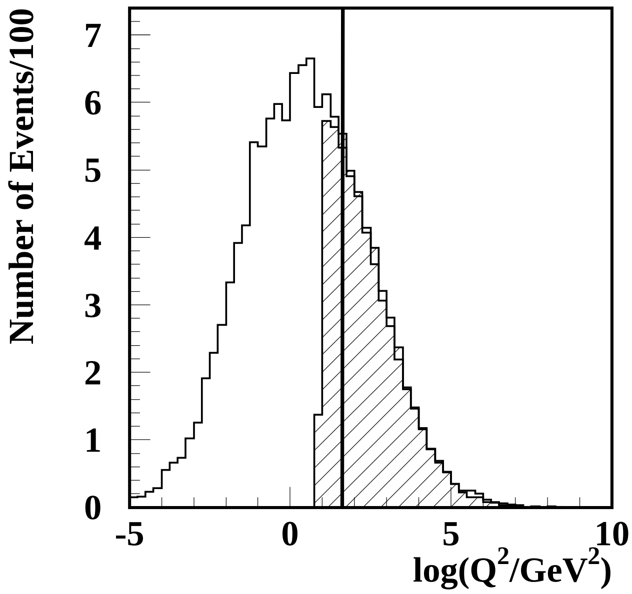


Figure 2.7: Distribution of $log\left(\frac{Q^2}{GeV^2}\right)$. The open histogram represents the inelastic mode, the dashed histogram the DIS mode. The solid line indicates the cut used to separate the two modes.

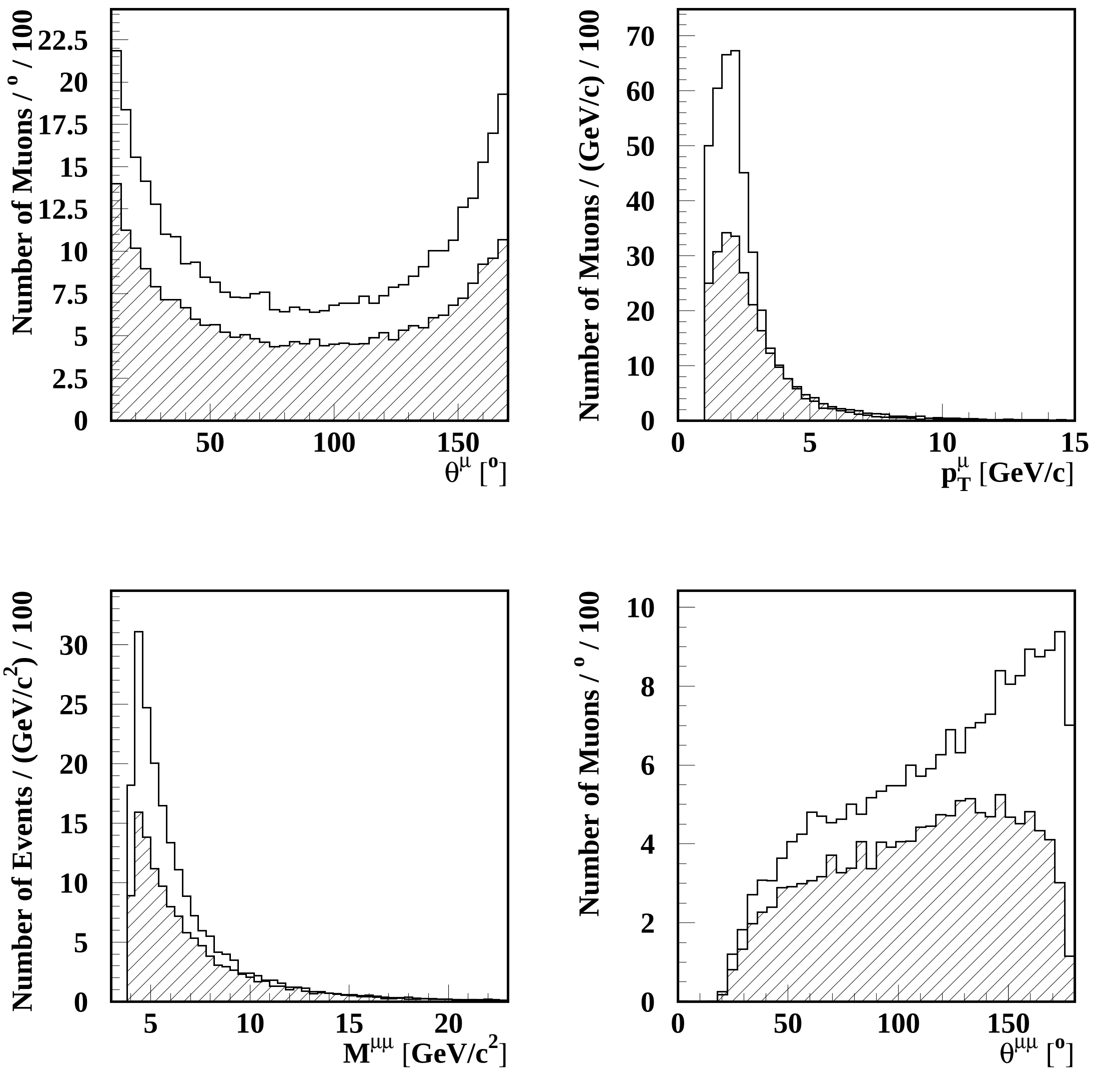


Figure 2.8: Distribution of polar angle and transverse momentum of the muons, invariant mass and opening angle of the two muons. The open histogram shows elastic, the dashed one inelastic muon production. The events are generated with LPAIR++ with the cuts: $M^{\mu\mu}$ >4 GeV/c$^2$, $p_T^\mu > 1.0$ GeV/c, $10° < \theta^\mu < 170°$.

# Chapter 3

# Experimental Setup

## 3.1 The HERA Collider

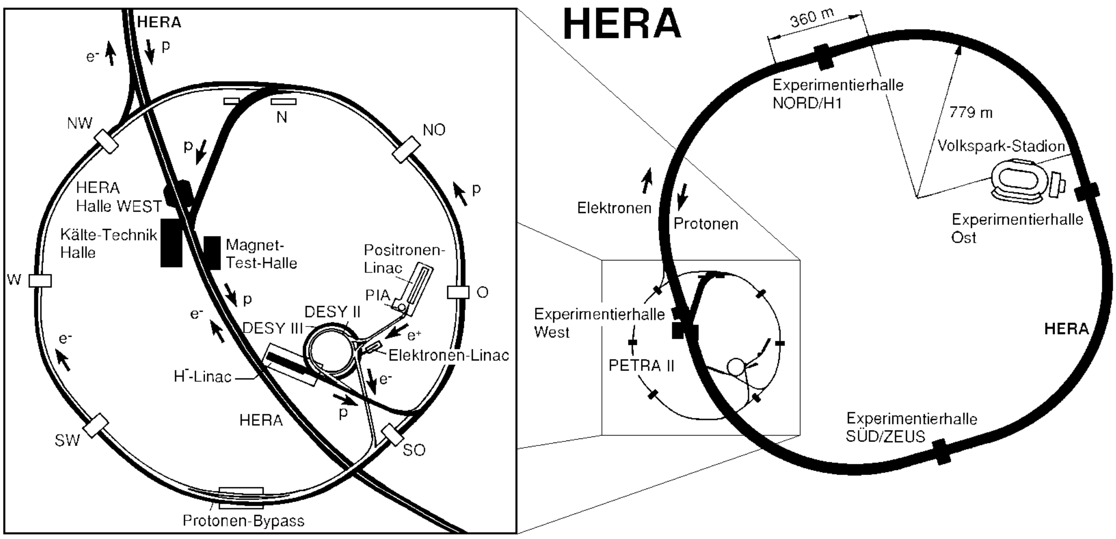


Figure 3.1: The accelerators at DESY

HERA (**H**adron **E**lectron **R**ing **A**nlage) is the first storage ring where electrons (here, electrons refers to both charge states, electrons and positrons) are brought to collision head on with protons. It is situated at the DESY laboratory in Hamburg and is in operation since 1992. Figure 3.1 shows a sketch of the ring and its preaccelerators. Protons and electrons are accelerated in Linac, DESY and PETRA up to an energy of 40 GeV and 14 GeV respectively. Then they are injected into HERA, where they are further accelerated to the final energy of 820 GeV and 27.5 GeV respectively. This corresponds to a center of mass energy of 300 GeV. Table 3.1 summarizes some parameters of the HERA collider.

| Design parameter | | | | |
|---|---|---|---|---|
| | p | | $e$ | |
| Circumference | 6336 m | | | |
| CMS energy | 300 GeV | | | |
| Luminosity | 1.5 $10^{31}$ $cm^{-2}s^{-1}$ | | | |
| Bunch length $\sigma_z$ | 85 mm | | 9 mm | |
| Beam width $\sigma_x$, $\sigma_y$ | 0.3 mm, 0.05 mm | | 0.3 mm, 0.05 mm | |
| Energy loss per turn | 6.2 eV | | 127 MeV | |
| **96/97 running periods** | | | | |
| | 1996 | | 1997 | |
| | p | $e^+$ | p | $e^+$ |
| Energy | 821.6 GeV | 27.6 GeV | 821.1 GeV | 27.5 GeV |
| Av. initial currents | 65 mA | 32 mA | 77 mA | 36 mA |
| Peak Luminosity | $8.3 \cdot 10^{30}$ $cm^{-2}s^{-1}$ | | $8.4 \cdot 10^{30}$ $cm^{-2}s^{-1}$ | |

Table 3.1: Some HERA collider parameters

At two points along its circumference of 6336 m, the two beams are brought to collision. Two general purpose detectors (H1 and ZEUS) are built around these beam crossing points. Up to 210 bunches of protons and electrons can be stored. This leads to a frequency of bunch crossings of 10.4 MHz or a time interval between two bunch crossings of 96 ns.
Two more experiments are situated at HERA. HERMES uses the lepton beam only. It is investigating the spin structure of the proton. The other experiment, HERA-B, uses the proton beam only. It wants to measure CP violation in B meson decays and will soon start taking data.

## 3.2 The H1 Detector

Figure 3.2 shows a sectional drawing of the H1 detector [7] together with the H1 coordinate system. The origin of the coordinate system is in the nominal interaction point. The positive z-axis points in the direction of the proton beam and is referred to as *forward* direction. Accordingly, the *backward* direction is the direction of the lepton beam. As indicated in the figure, the polar and azimuthal angles $\theta$ and $\phi$ are defined relative to the forward direction and the x-axis respectively.
The detector is much more massive and more elaborately instrumented in the forward direction. This is necessary, because the $e-p$ center of mass system is boosted along the forward direction with $\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 2.86$.