The Determination of the Strong Coupling and of the Gluon Density Using Deep Inelastic $(2+1)$ Jet Events at HERA

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Abstract

The analysis methods and first results of the determinations of the strong coupling parameter α_s and the gluon density in the proton from the measurement of $(2+1)$ jet events in deep inelastic scattering at HERA, in a kinematic region previously not accessible, are reviewed. Experimental and theoretical problems which need to be addressed are identified.

Résumé

Les méthodes d'analyse et prémiers résultats expérimentaux sont présentés pour extraire la couplage de l'interaction forte α_s et la densité du gluon dans le proton en mésurant événements à $(2+1)$ jets dans un nouveau domaine cinematique de la diffusion profondément inélastique accessible à HERA. Les problémes expérimentaux et théoriques, qui doivent être traités dans l'avenir, sont exposés.

1. Introduction

Results from the H1 and the ZEUS collaboration on the strong coupling parameter $\alpha_s(Q^2)$ and the gluon density $x_g \cdot g(x_g, Q^2)$ of the proton are presented and discussed. These results are derived from measurements of the (2+1) jet cross sections in deep inelastic scattering (DIS). The tree level graphs for boson gluon fusion (BGF) and QCD-Compton (QCDC), leading to the production of $(2+1)$ jet events, are shown in figure 1. In leading order (LO) the $(2+1)$ jet cross section can be written as

$$\sigma_{2+1} \sim \alpha_s(A\, g(x_g, Q^2) + B\, q(x_q, Q^2)) \qquad (1)$$

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with $A, B \sim f(x, Q^2, x_p, z_p, \phi^*)$. The variable x is

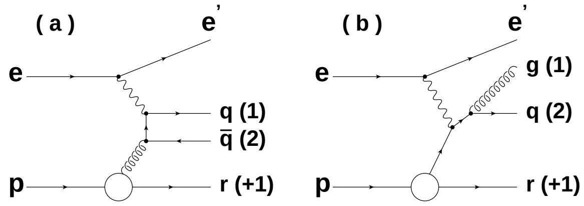

Figure 1. *Tree graphs for a) BGF and b) QCDC (crossed graphs are not shown).*

the Bjorken scaling variable and Q^2 the negative 4-momentum transfer squared of the virtual photon. Instead of the energy, polar and azimuthal angle of one of the final partons of the tree level processes, the more convenient variables $x_p = x/x_i$, z_p (see later), and ϕ^* are used. Here x_i is the momentum fraction of the incoming parton ($i = g, q$, or $\bar{q}$) and ϕ^* its azimuthal angle with respect to the lepton scattering plane in the γ^*-proton center-of-mass system (hadronic cms).

Now obviously, in order to determine α_s, one has to choose a region of phase space where the parton densities $g(x_g, Q^2)$ and $q(x_q, Q^2)$ are well known. This implies $x_g > 10^{-2}$ and $x_q \geq 10^{-3}$ to 10^{-2}. In this kinematic domain at HERA, without any additional cut on the minimal Q^2, the cross sections for BGF and QCDC are of similar size. To determine α_s at some Q^2 one counts the number of $(2+1)$ jet events normalized to for example all events. This is very similar to what is done in e^+e^- annihilation. Yet in DIS there is an important difference: strongly interacting particles are already found in the initial state.

A first and simple determination of the gluon density requires, as can be seen from eq.1, knowing α_s and the quark densities. To minimize the importance of the uncertainties of the latter one chooses the region of low Q^2 and low x, where $\sigma_{BGF} \sim (3-4)\,\sigma_{QCDC}$.

In addition, it has become clear in the course of the analyses that one has to exclude jets in the very forward region (direction of the incident proton) for two reasons, one theoretical and one experimental. According to the data, there is significant jet production in the forward region due to initial state parton showers, which cannot be described by fixed second order calculations used for the extraction of α_s. Experimentally, the measurement of jets in the forward direction is difficult due to lack of acceptance because of the beam-pipe and due to the thickness of inactive materials at small angles.

2. The determination of the strong coupling

Both experiments, H1 and ZEUS, determine the rate of $(2+1)$ jet events using the JADE jet algorithm [1] modified for ep interactions by including a "pseudo-particle", represented by the missing longitudinal momentum in the detector, as best estimate of the "invisible" proton-remnant. Pairs of "particles" (calorimetric energy deposits and the proton-remnant) are combined until all remaining pairs have an invariant mass squared $m_{ij}^2 > y_{\rm cut} W^2$. W is the invariant mass of the hadronic system. The jet rate depends on $y_{\rm cut}$, the jet resolution parameter. This algorithm appears to be quite suitable, but it is also the only one for which the necessary next to leading order (NLO) calculations exist [2, 3, 4] so far.

I will be brief in reporting the results from the H1 experiment, since they have been published already [5]. Two data samples, taken in 1993, with an integrated luminosity of about 0.3 pb^{-1} were analyzed. A low Q^2 sample with $10 < Q^2 < 100$ GeV^2 consisting of ≈ 12500 events and a high Q^2 sample with $Q^2 > 100$ GeV^2 containing ≈ 740 events. In order to suppress forward jets, the polar angle of reconstructed jets ($\theta_{\rm jet}$) had to be within $10°$ and $145°$, where θ is defined with respect to the proton beam direction ($+z$ axis). Using both samples, H1 showed that a constant α_s (in contrast to a running coupling), in the range of Q^2 of the experiment, does not give a good description of the data, independently of which Monte Carlo (MC) model was used for the corrections.

In the determination of α_s, the $(2+1)$ jet rate is corrected back to the level of partons for which NLO calculations exist. The corrections include effects of the detector acceptance and resolution, QED radiation, and hadronisation. They were estimated using LEPTO 6.1 [6] and a complete detector simulation, once with the option (MEPS) of generating exact zeroth and first order matrix elements (ME), with additional parton showers (PS) according to the leading log approximation, and once with the option of the color dipole model [7] as implemented in ARIADNE 4.03 [8]. In the latter case LEPTO generates the neutral current cross section and the BGF matrix element, and ARIADNE the QCDC matrix element and the parton shower in the color dipole approximation. For the low Q^2 sample, the corrections to the jet rate, derived from the two models, differed considerably, approaching each other, however, as Q^2 increased.

For this reason, the low Q^2 data were excluded from the determination of a precise value for α_s at $Q^2 = M_Z^2$. Using the two highest Q^2 points ($Q^2 > 100$ GeV^2 sample), H1 obtained the following result: $\alpha_s(M_Z^2) = 0.123 \pm 0.012$ (stat.) ± 0.013 (syst.) [5]. The dominant systematic uncertainties are due to the QCD models for the correction (0.008) and the hadronic energy scale of the calorimeter (0.006), both of which are expected to decrease considerably with more data and the ensuing better understanding.

The ZEUS collaboration also published results on jet production [9] using their 1993 data. They analyzed data with an integrated luminosity of about 0.55 pb^{-1} resulting in ≈ 1020 events with $160 < Q^2 < 1280$ GeV^2, $0.01 < x < 0.1$, and $0.04 < y < 0.95$. Here y is the fractional energy transfer between the electron and proton in the proton rest frame. The motivation for the requirement $x > 0.01$ is twofold, to reduce the contribution from initial state parton showers and to reduce the impact of the uncertainties in the parton densities on the $(2+1)$ jet event rate. This is demonstrated in figure 2, where a) the ratio of the rate of (2+1) jet events from LEPTO using either MEPS or ME only is shown and b) the same ratio is evaluated using either the parton density function MRSD$'_-$ or MRSD$'_0$ [10]. The first function has at small x a very steeply rising gluon distribution for decreasing x, whereas the

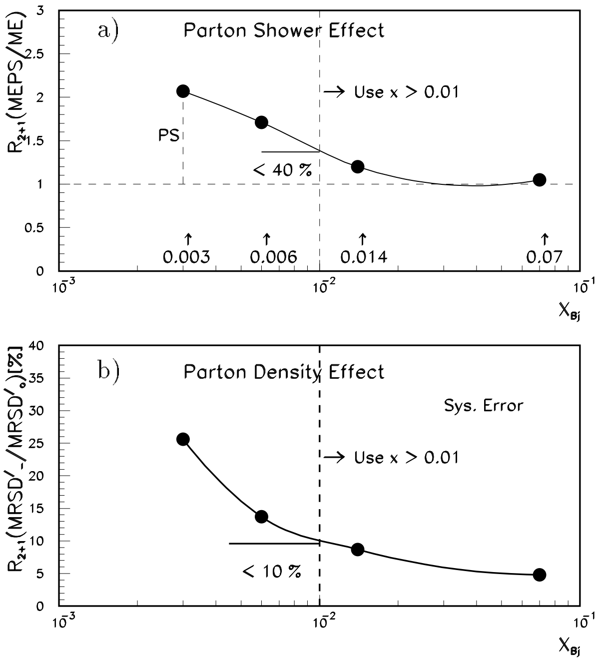

Figure 2. *Effect of a) parton showers and b) the parton density parametrisations* $MRSD'_-$ *and* $MRSD'_0$ *(ZEUS).*

behaviour is rather flat for the second function.

No value for α_s was extracted, however, since (with these cuts) a significant discrepancy between data and MC, i.e. with LEPTO (MEPS) and with the NLO program PROJET [4] was observed. This can be seen in Fig. 8b and 8d of reference [9], where the dependence of the normalized number of $(2+1)$ jet events on z_p is shown. The invariant $z_p = (P \cdot p_{\rm jet})/(P \cdot q)$ is in the γ^*-parton center-of-mass system given by $1/2(1 - \cos\tilde{\theta}_{\rm jet})$. P is the 4-momentum of the proton, $p_{\rm jet}$ of one of the outgoing jets, and q of the virtual photon. The discrepancy at small values of z_p is due to an excess of forward jet events in the data compared to MC. This excess is reflected in the x_p distribution in Fig. 8c [9].

A preliminary analysis of $\approx 3.3\ \mathrm{pb}^{-1}$ of data, taken in 1994, has confirmed the 1993 analysis. In order to remove forward jets, the cut on y was increased from 0.04 to 0.1 and z_p was restricted to lie within 0.1 and 0.9. With these cuts, the distributions of the partonic scaling variables x_p and z_p are found to be in good agreement with the MEPS model, as can be seen in figure 3. Good agreement between data and MEPS is also observed in the distributions of the pseudo-rapidities η of the forward and backward jets shown in figure 4.

The jet rates as a function of $y_{\rm cut}$, corrected to the parton level using LEPTO MEPS, are now found to be in good agreement with DISJET [3] and PROJET [4], two NLO calculations. One such comparison of data with DISJET for the bin $240 < Q^2 < 720\ \mathrm{GeV}^2$ is shown in figure 5.

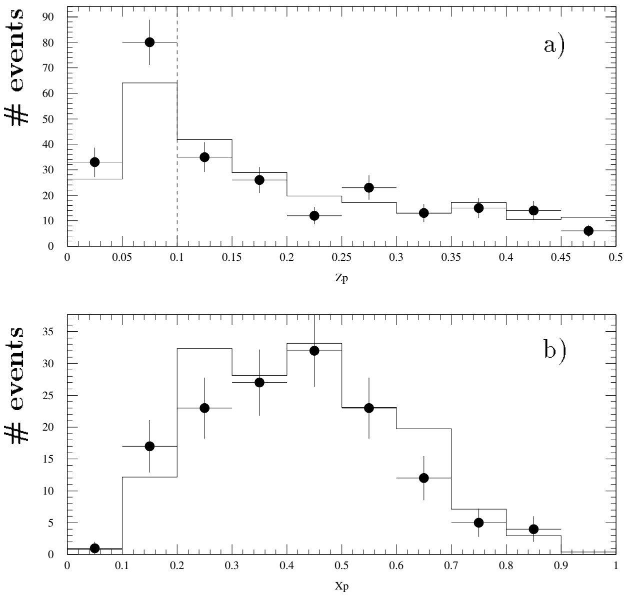

Figure 3. *Uncorrected a)* z_p *and b)* x_p *distributions compared to MC simulation based on the MEPS model (ZEUS preliminary).*

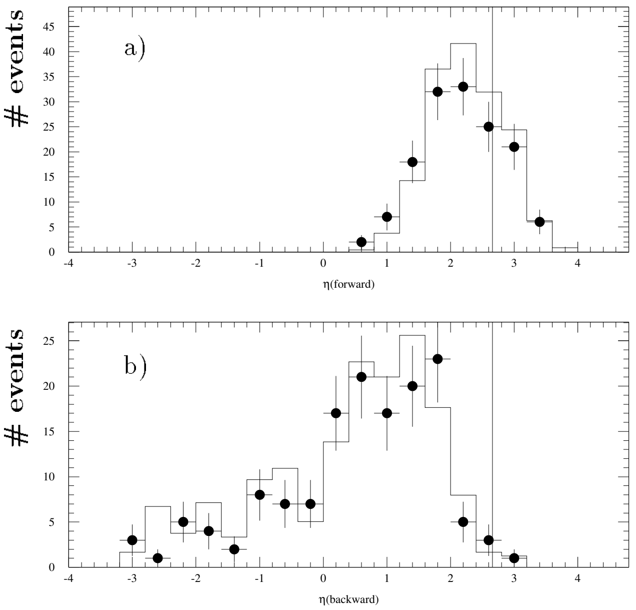

Figure 4. *Uncorrected* η *distribution of the a) forward and b) backward jets compared to MC simulation based on the MEPS model (ZEUS preliminary).*

Preliminary results from ZEUS on $\alpha_s(Q)$ have been derived from such comparisons and are shown in figure 6 together with QCD predictions for three different values of $\Lambda^{(5)}_{\overline{\rm MS}}$.

One can expect that final analyses by H1 and ZEUS of the 1994 data on (2+1) jet events for $Q^2 > 100\ \mathrm{GeV}^2$ will yield a determination of $\alpha_s(M_Z)$ with errors similar

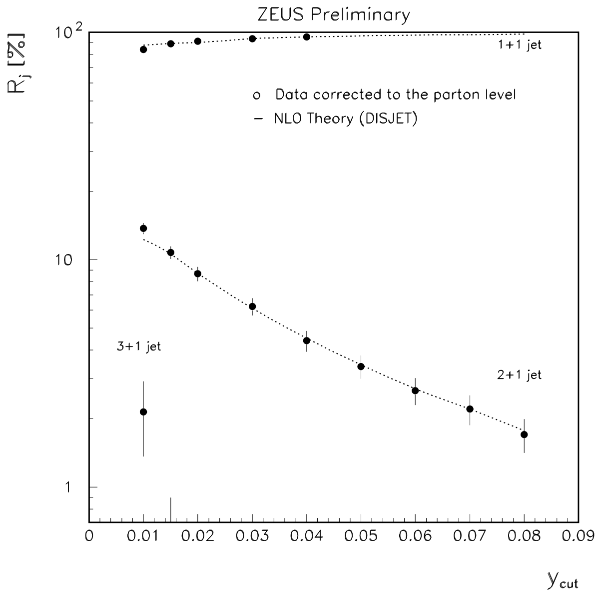

Figure 5. *Jet rates (corrected to the parton level) as a function of the jet resolution parameter $y_{\rm cut}$ compared to DISJET, a NLO calculation (ZEUS preliminary).*

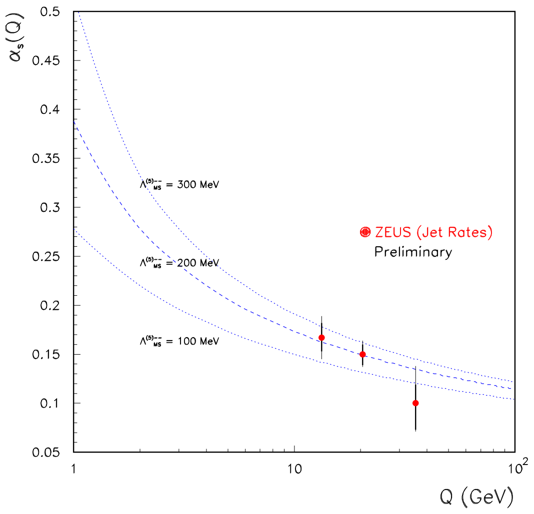

Figure 6. *$\alpha_s(Q)$ measurements from $(2+1)$ jet rates (ZEUS preliminary), compared to the QCD predictions for $\Lambda^{(5)}_{\overline{\rm MS}} = 100$, 200, and 300 MeV.*

to the corresponding results from LEP. This will be an important test of QCD.

3. The determination of the gluon density

Open $c\bar{c}$ production comes to mind, when thinking about a direct determination of the gluon density using $(2+1)$ jet events. The $c\bar{c}$ tags BGF events. Using however primarily light flavour $(2+1)$ jet production has the advantage of a much larger cross section compared to heavy flavour production. The benefit of DIS is a negligible background from resolved photoproduction processes. Of disadvantage are of course the backgrounds due to the QCDC process and due to migrations from the lowest order quark parton model (QPM) process and higher order processes into the class of $(2+1)$ jet events.

The analyses of both experiments presented here are based on their 1993 data. The H1 results have become final after this conference [11], the ZEUS ones are still preliminary. The selection criteria for the DIS events and the jets of both experiments are summarised in table 1. In both analyses jets are reconstructed

	H1	ZEUS
$E_{e'}[GeV]$	$> 10.$	$> 10.$
Q^2 [GeV^2]	$12.5 < Q^2 < 80.$	$10. < Q^2 < 100.$
y	$y_e > 0.05$ †	$y_h > 0.05$
$\Delta R = \sqrt{\Delta\eta^2 + \Delta\Phi^2}$	1.	1.
$E^*_{T,\rm jet}$ [GeV]	> 3.5	$> 4.$
$\eta^*_{\rm jet}$		< 0.5
$\theta_{\rm jet}$ (LAB)	$10^\circ < \theta_{\rm jet} < 150^\circ$	
$\Delta\eta_{\rm jet}$ (LAB)	$< 2.$	
$\hat{s}$ [GeV^2]	$> 100.$	$> y_{\rm min} W^2$
$\lvert\Delta\sqrt{\hat{s}_{\rm a,b}}\rvert$ [GeV]	$< 10.$	

Table 1. *Event selection and jet cuts for the analysis of the gluon density .*

in the hadronic cms using a cone algorithm [12] ‡. The cone size ΔR is calculated from $\Delta\eta$ and $\Delta\Phi$ (see table 1), which are the differences in pseudo-rapidity and azimuthal angle between two calorimetric energy clusters.

For events with $(2+1)$ jets, the invariant mass squared $\hat{s}$ of the hard two jet subsystem can be reconstructed. H1 applied two methods: a) making use of the energy deposits and their directions, measured with the calorimeters, which belong to the two jets:

$$\hat{s} = (\sum_j p_j)^2 \tag{2}$$

and b) using the directions of the two jets in the hadronic cms:

$$\hat{s} = W^2 \exp^{-(\eta_1^* + \eta_2^*)} \tag{3}$$

where $\eta^*_{1,2}$ are the pseudo-rapidities of the two jets and W is the invariant mass of the hadronic system. The ZEUS analysis uses method a). The main difference

† For the measurement of y_e (y_h) the scattered lepton (hadronic system) is used.
‡ H1 has also used the modified JADE algorithm in the laboratory frame and has obtained within statistical errors the same result for the gluon density.

between the H1 and ZEUS analyses is that H1 requires a fixed minimum $\hat{s}$, while in the ZEUS analysis $\hat{s}$ scales with $y_{\min} W^2$ ($y_{\min}$ will be discussed later). In addition, H1 demands that the difference between the corrected (for effects of the detector and jet definition) $\hat{s}$ values, reconstructed with methods a) and b), be small (see table 1).

With these requirements both experiments observe clear jet structures. This is shown for H1 in figure 7 which displays the transverse energy flow in the laboratory frame as a function of $\Delta\Phi$ and $\Delta\eta$, for the most backward going jet in a) and b), and for the most forward going jet in c) and d), using the respective jet axis as reference. The angle Φ is measured away from the scattered electron for the jet at the smallest η. The points represent the $(2+1)$ jet data and the errors given are statistical. The histograms give the predictions of the MEPS model which is in very good agreement with the data.

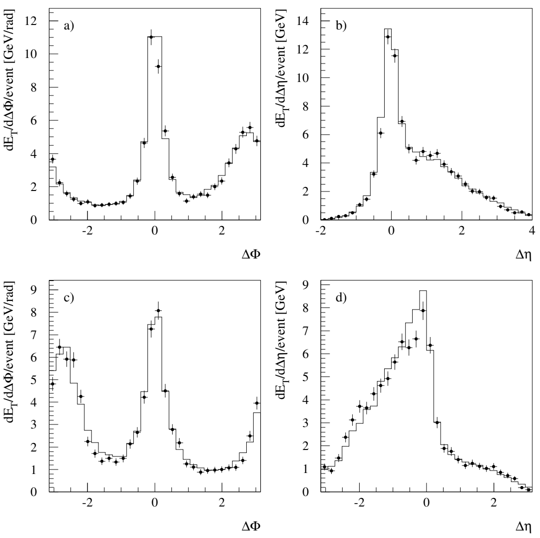

Figure 7. *Transverse energy flow for the $(2+1)$ jet events compared to MC simulation based on the MEPS model (H1).*

Having determined x and Q^2 from the measurement of the scattered electron and $\hat{s}$ from the two jets, x_g can then be calculated using the relation:

$$x_g = x\,(1 + \hat{s}/Q^2). \tag{4}$$

The LEPTO MEPS model was used to investigate the correlation between x_g as calculated from the hard partons originating from the BGF matrix element and $x_g^{\rm rec}$ as determined from the reconstructed jets after detector simulation. The relative errors $\Delta x_g/x_g = (x_g^{\rm rec} - x_g)/x_g$ in the reconstruction of x_g, using methods a) and b) for the calculation of $\hat{s}$, are shown in figure 8. Method b) results in a superior resolution but exhibits a more pronounced tail. For properly reconstructed jets both methods should give consistent results. A misassignment of particles to the jets is expected to have different impacts on $x_g^{\rm rec}$, depending on the two methods of reconstructing $\hat{s}$. As shown in figure 8c), an improved resolution can be observed for those events for which the results of the two methods agree within the resolution. For an event satisfying the cut on $|\Delta\sqrt{\hat{s}_{\rm a,b}}|$, the mean value of the reconstructed $\hat{s}$ values gives the "combined" x_g. About 20% of the events are removed due to this cut. In figure 8d) finally the correlation between the reconstructed and the true x_g is shown for the combined method.

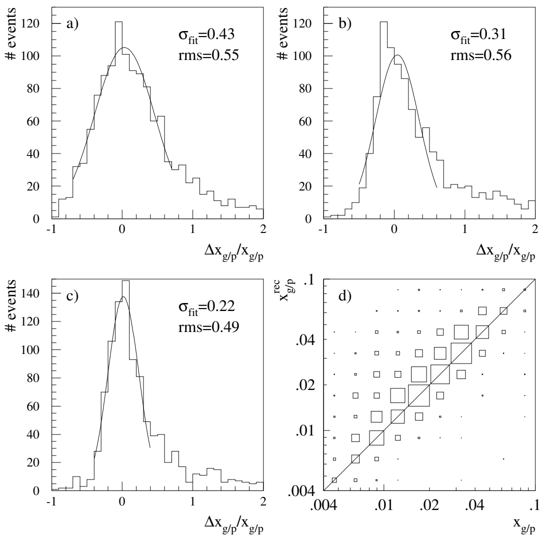

Figure 8. *The relative error in the reconstrucion of x_g †and the correlation between the reconstructed and the true x_g for BGF events from a MC simulation based on the MEPS model (H1).*

Unfolding procedures with regularisation [13, 14] were used by both experiments to determine the gluon density. The MEPS model was used to calculate in LO the contributions from BGF events and from background QCDC events and QPM events, including leading log parton showers, to the observed number of $(2+1)$ jet events. Detector effects were unfolded using a complete detector simulation. In the LEPTO MC the region of phase space where the QCD matrix elements diverge, for soft and/or collinear emissions of partons, is avoided by a requirement on the smallest invariant mass m_{ij} between any two partons, including the proton remnant. Since it is of interest to measure the gluon density down to as small an x_g as possible, one chooses

† Both x_g and $x_{g/p}$ refer to the fractional momentum of the gluon in the proton.

$\hat{s}$ (and therefore m_{ij}) as small as experimental jet finding and diverging matrix elements allow.

In the ZEUS analysis $m_{ij}^2 > y_{\rm min} W^2$ is used, which is the default requirement in LEPTO. Consequently (2+1) jet events have to fulfill $\hat{s} > y_{\rm min} W^2$. The cut-off $y_{\rm min}$ is then varied between 0.0025 and 0.01 to study its influence on the gluon density. The x_g distributions of the data and the MEPS model are plotted in figure 9, for three different values of $y_{\rm min}$. For MEPS the individual contributions from BGF, QCDC, and QPM with parton showers are indicated.

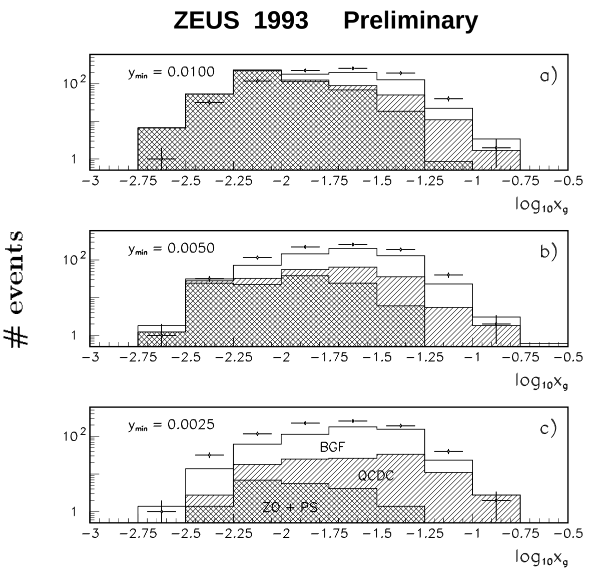

Figure 9. *Comparison of x_g distributions from data and MC simulation based on the MEPS model for different $y_{\rm min}$ (ZEUS preliminary).*

The H1 collab. has modified the LEPTO MC to use a parametrisation for the cut in m_{ij}, which everywhere in phase space is 2 GeV above where the O(α_s) (2+1) jet cross section would exceed the total cross section [11]. This cut-off is always significantly below 10 GeV which is the cut applied in the event selection of reconstructed data and MC. Therefore the full phase space used in this analysis is covered by the matrix elements.

The unfolded LO gluon densities from H1 and ZEUS are shown in figure 10, for an average Q^2 of 30 GeV^2. The error bars indicate the statistical and sytematic errors. Not included is a global normalisation uncertainty, estimated to be 11% for H1, due to uncertainties in the value of α_s, the luminosity measurement, and detector efficiencies. The data from both experiments agree within errors, however, the systematic errors currently estimated by ZEUS are considerably larger. This is mainly due to a large contribution to the error from the variation of $y_{\rm min}$. In the case of H1 the dominant contribution to the

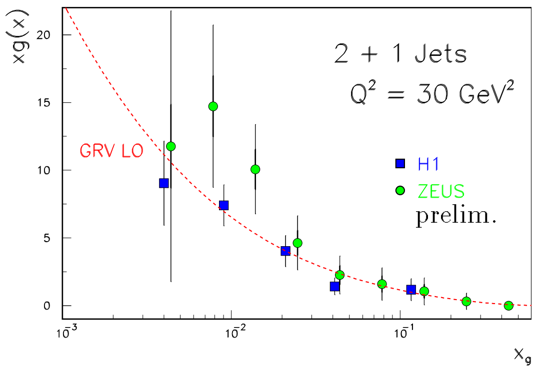

Figure 10. *The gluon density in the proton as a function of x_g, determined in LO from the rate of $(2+1)$ jet events. Shown are data from H1 and ZEUS (preliminary), and the LO GRV [15] gluon density parametrization.*

systematic error at small x_g is 13%, coming from the uncertainty in the hadronic energy calibration of the liquid argon calorimeter, while at large x_g it is 35%, due to the variation of the $\Delta\sqrt{\hat{s}_{\rm a,b}}$ cut.

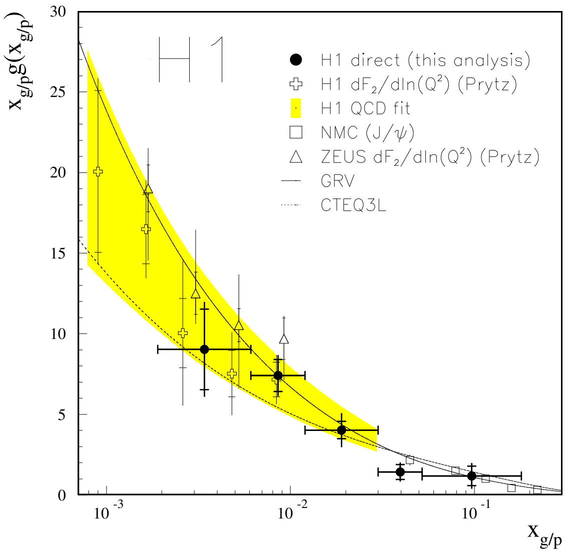

Figure 11. *The gluon density in LO by H1 at an average Q^2 of 30 GeV^2 compared with indirect determinations by H1 and ZEUS at $Q^2 = 20$ GeV^2, direct results from J/ψ by NMC evolved to $Q^2 = 30$ GeV^2, and the GRV and CTEQ3L parametrisations.*

These measurements provide the first direct determinations of the gluon density in LO at such low x_g values. Indirect constraints on the gluon density for low x_g values have been derived from the Q^2 evolution of the quark densities. In figure 11 the direct measurement from H1 [11] is compared with recent indirect results from H1 [16] and ZEUS [17], extending to even lower x_g values. Also shown are direct measurements from NMC [18] for $x_g > 0.04$. They are based on J/ψ production in DIS.

The data in figure 11 are also compared to two different parametrisations (GRV [15] and CTEQ3L [19]) of the gluon density in LO. Both are based on the Altarelli-Parisi evolution equations. The GRV model assumes the gluon and quarks distributions to be valence-like at $Q^2 = 0.23$ GeV^2. The growth of the gluon density with decreasing x_g is then due to the radiation of low x partons according to the evolution equations. The CTEQ3L parametrisation is based on input distribution functions at $Q^2 = 4$ GeV^2, assuming the sea and gluon densities to have the same power dependence of the fractional momentum x_i.

Within the errors one finds the indirect and direct determinations of the gluon densities in LO to be compatible. Strictly speaking this is not required by QCD in LO. Therefore it will be of great interest to compare analyses in NLO when they become available.

4. Conclusion and Open Points

First results on the determination of α_s in NLO and on the gluon density of the proton in LO have been obtained at HERA from measurements of (2 + 1) jet events. With more data to come, a better understanding of the details of the physics involved and of the detectors, and progress in theoretical calculations, the determination of $\alpha_s(Q^2)$ and $g(x_g, Q^2)$ from jet events in *ep* collisions will allow fundamental tests of QCD.

In particular, the measurement of α_s allows a quantitative comparison between QCD corrections to the scale-invariance of the parton distributions and properties of the accompanying hadronic final state. Of equal importance is the determination of α_s at different Q^2 and a comparison with the result from LEP based on jet counting. The measurement of the gluon density can be compared with results from scaling violations in inclusive DIS and heavy quark flavour production. At HERA we have just begun.

In order to make progress many problems have to be addressed. I offer here my list of the most urgent experimental and theoretical tasks:

- Improve the understanding of experimental questions related to detector response and calibration.
- Improve the understanding of the $Q^2 < 100$ GeV^2 region (forward going jets) and reduce the dependence of the MC corrections due to the different models.
- Provide NLO calculations of jet rates for other jet algorithms (k_T [20], cone [12, 21]) and a corresponding NLO Monte Carlo simulating the complete partonic and hadronic final state.
- Develop methods to determine the gluon density in NLO [22].

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