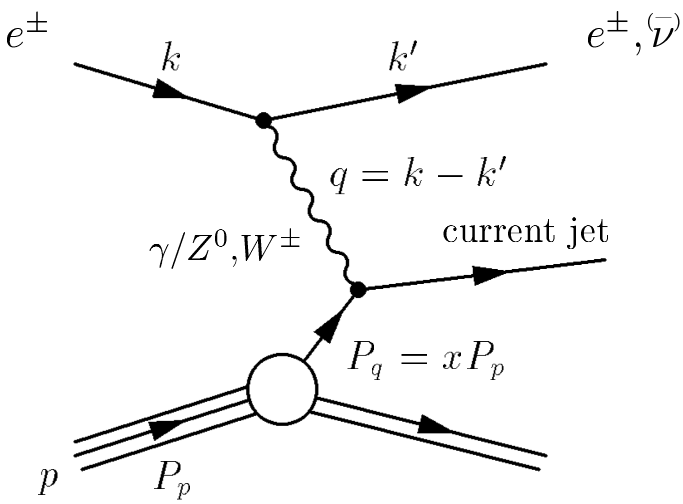
Low-x and Saturation

R. Yoshida
Argonne National Laboratory

DESY Seminar, 30 November, 2000

- **Proton Structure Function F_2 at Small-x**
- **Diffraction at HERA**
  - **Diffraction in DIS**
  - **Diffractive Vector Meson Production**
- **F_2 at Small-x Revisited**
- **Comments and Questions**

I. F_2 at Small-x



$Q^2 = -q^2$: virtuality ("size" of the probe)

x: mom. frac. of the struck parton (inf. mom. frame)

Experimentally, we measure (At $Q^2 \ll M_Z^2$, and ignoring F_l):

$$\frac{d\sigma^2}{dxdQ^2} \approx \frac{2\pi\alpha^2}{xQ^4}(1 + (1 - y)^2)F_2(x, Q^2)$$

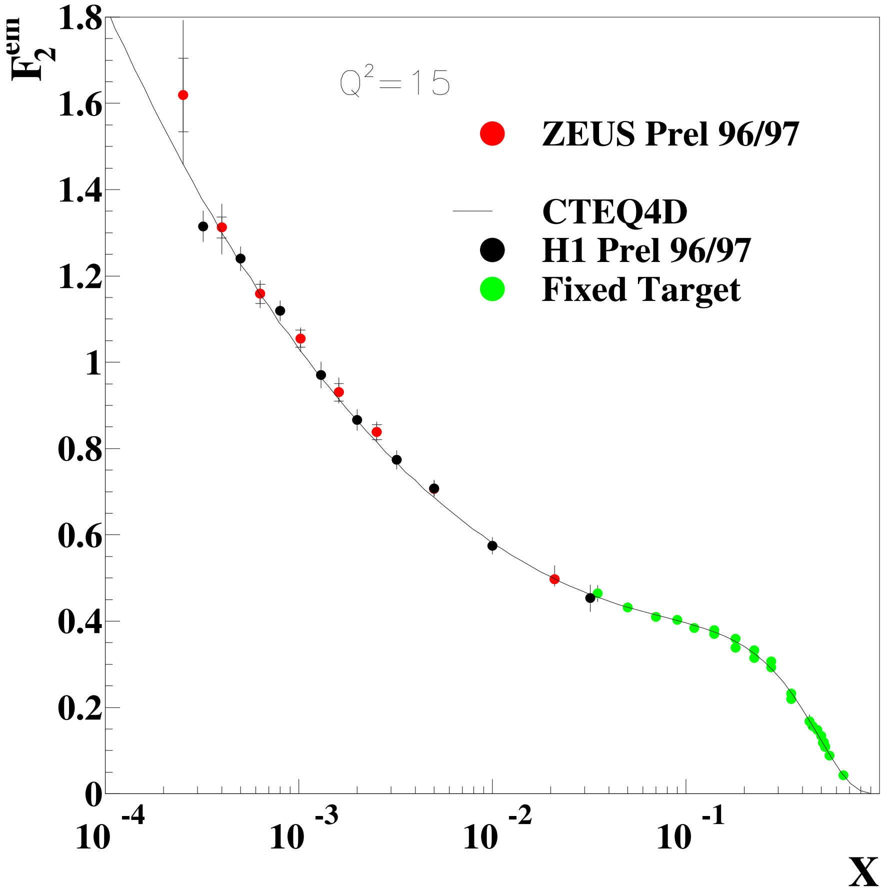
$y = Q^2/xs$, the inelasticity parameter

And (LO (or in DIS scheme..))

$F_2(x, Q^2) = x\sum_q e_q^2(q(x, Q^2) + \bar{q}(x, Q^2))$

where q, $\bar{q}$ are quark, antiquark PDF's.

F_2 rises at small-x..

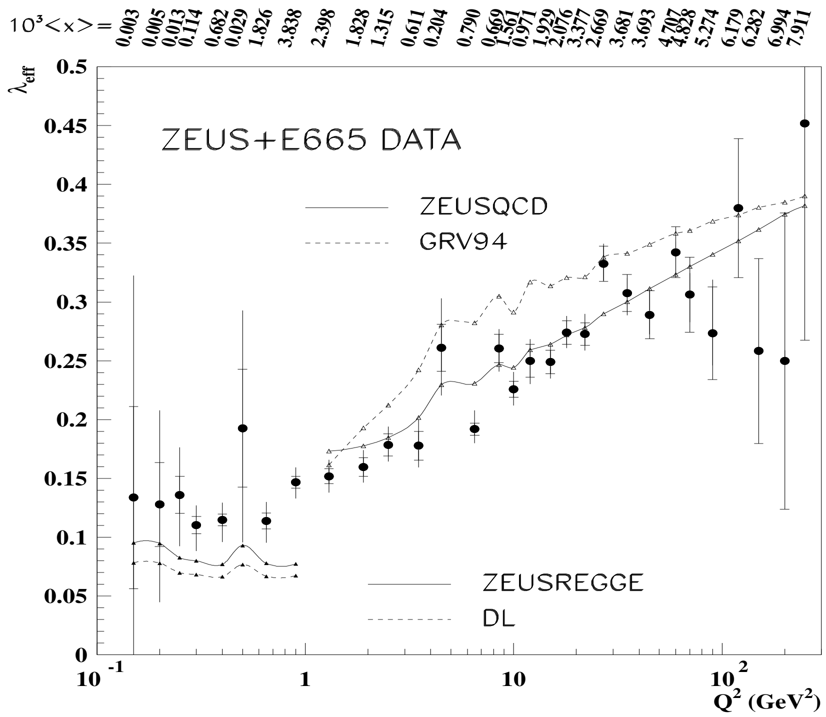


$\Rightarrow$ parton densities are rising at small-x Naively: valence quark $\rightarrow$ gluons $\rightarrow$ sea quarks $\rightarrow$ more gluons etc. etc.

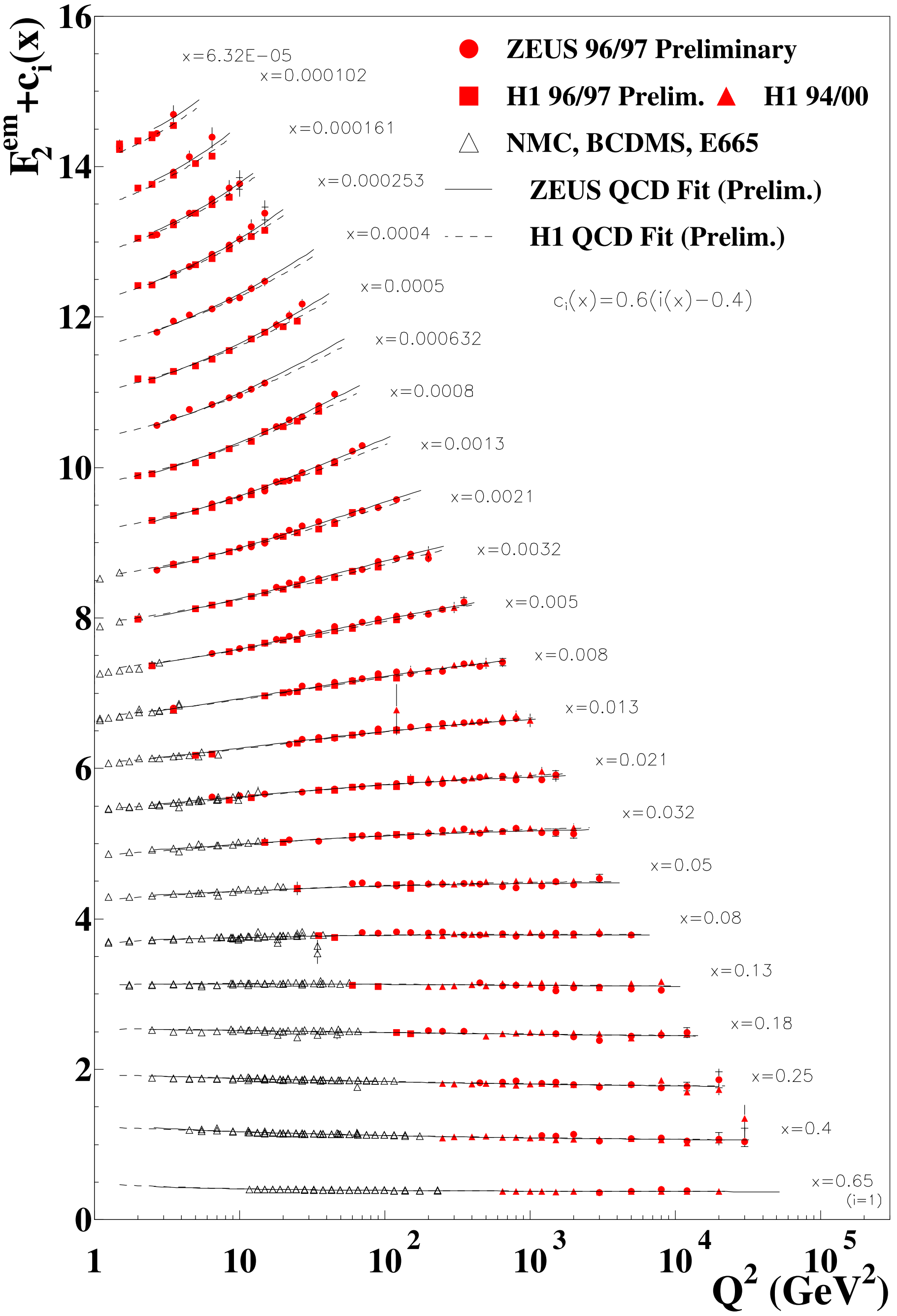
$F_2 \propto x^{-\lambda}$:

$\lambda \approx 0.2$

($Q^2 \approx 10$ GeV^2)



Proton Structure Function F_2

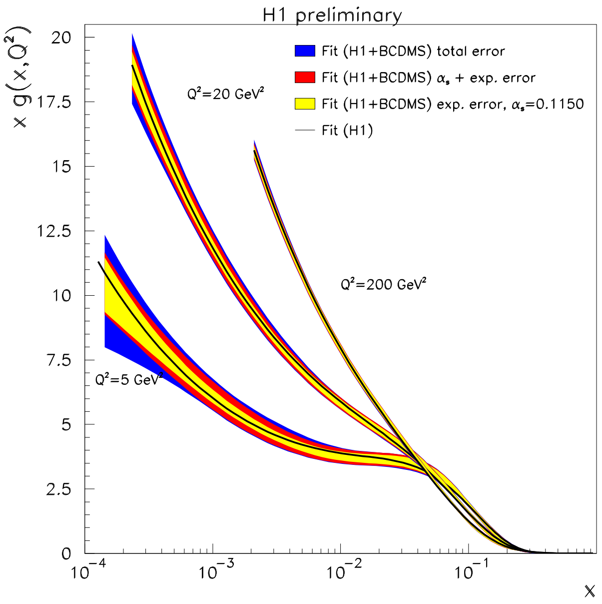


The GLAP fits describe the data well $Q^2 > 1$ GeV

Additive constant c_i added for display purpose

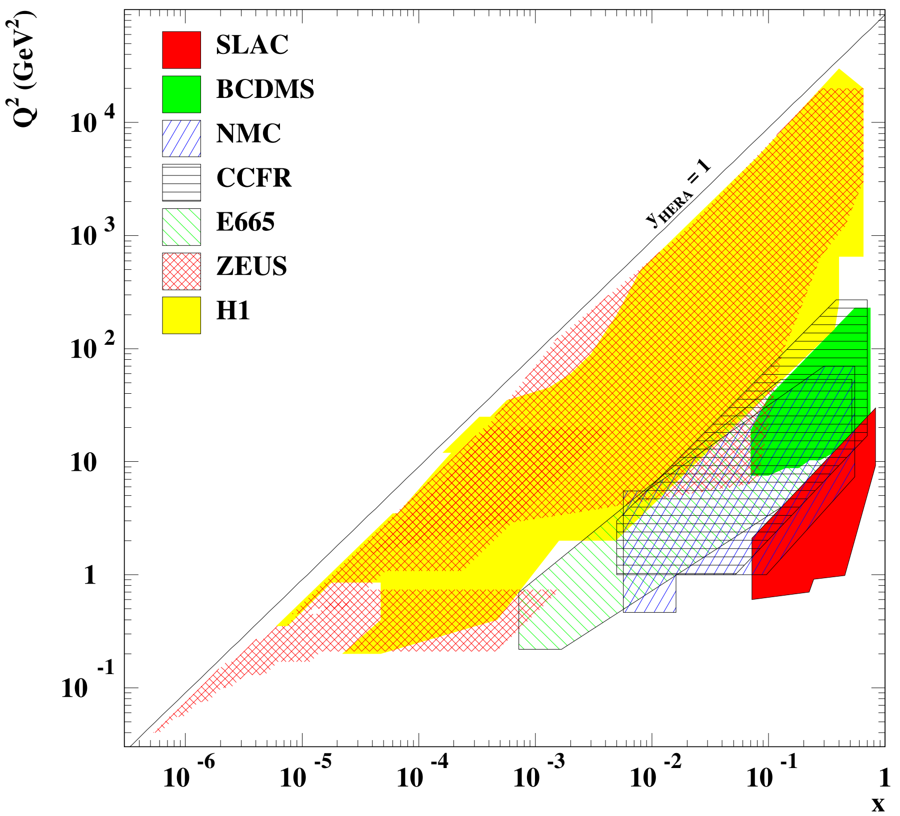
In LO: $\frac{\partial F_2}{\partial \ln Q^2} \sim \alpha_s x g(x, Q^2)$

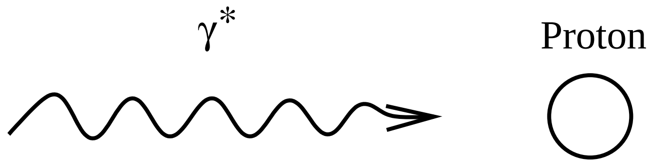
Gluons from a NLO GLAP fit to the F_2 measurements



Gluons rise at low-x as expected. (More on DGLAP fit studies from Max Klein.)

What about really low Q^2 (and low-x)?





Note: Small-x $\Rightarrow$ low Q^2 (Well below 1 GeV^2 at lowest x)

DIS IN THE HADRONIC PICTURE

$W^2 \approx Q^2/x$: the $\gamma^* p$ cms energy squared ("s")

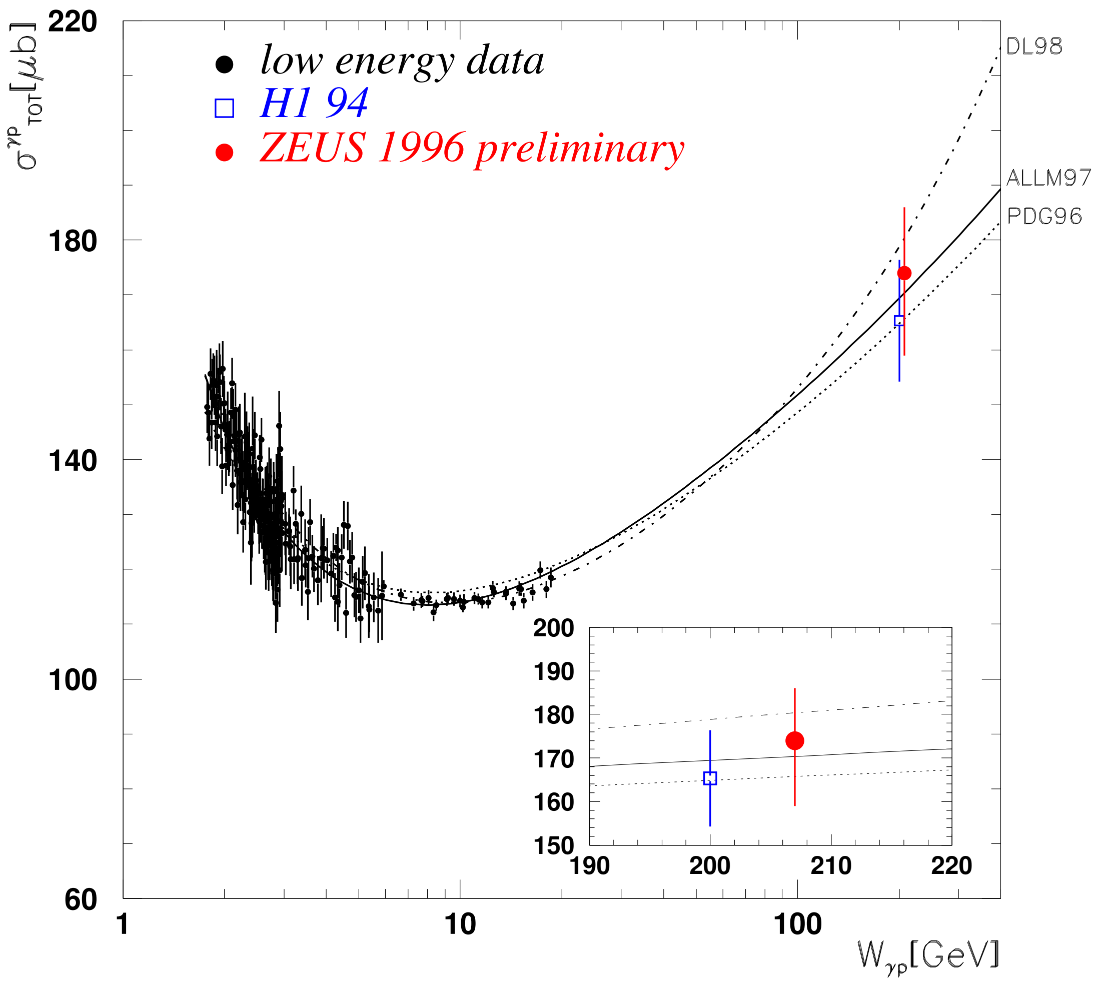
For fixed Q^2: $x^{-\lambda} \to W^{2\lambda}$ ($F_2 \sim W^{0.4}$, $Q^2 \sim 10$ GeV^2)

And at small-x:

$$\sigma_{tot}^{\gamma^* p}(W^2, Q^2) \approx \frac{4\pi^2\alpha}{Q^2} F_2(x \approx Q^2/W^2, Q^2)$$

- F_2 vanishes like Q^2 at low Q^2 (conservation of EM current):
- $\sigma_{tot}^{\gamma p}(W^2)$ described by Regge $[\sim W^{2(\alpha_{\mathbb{P}}-1)}]$

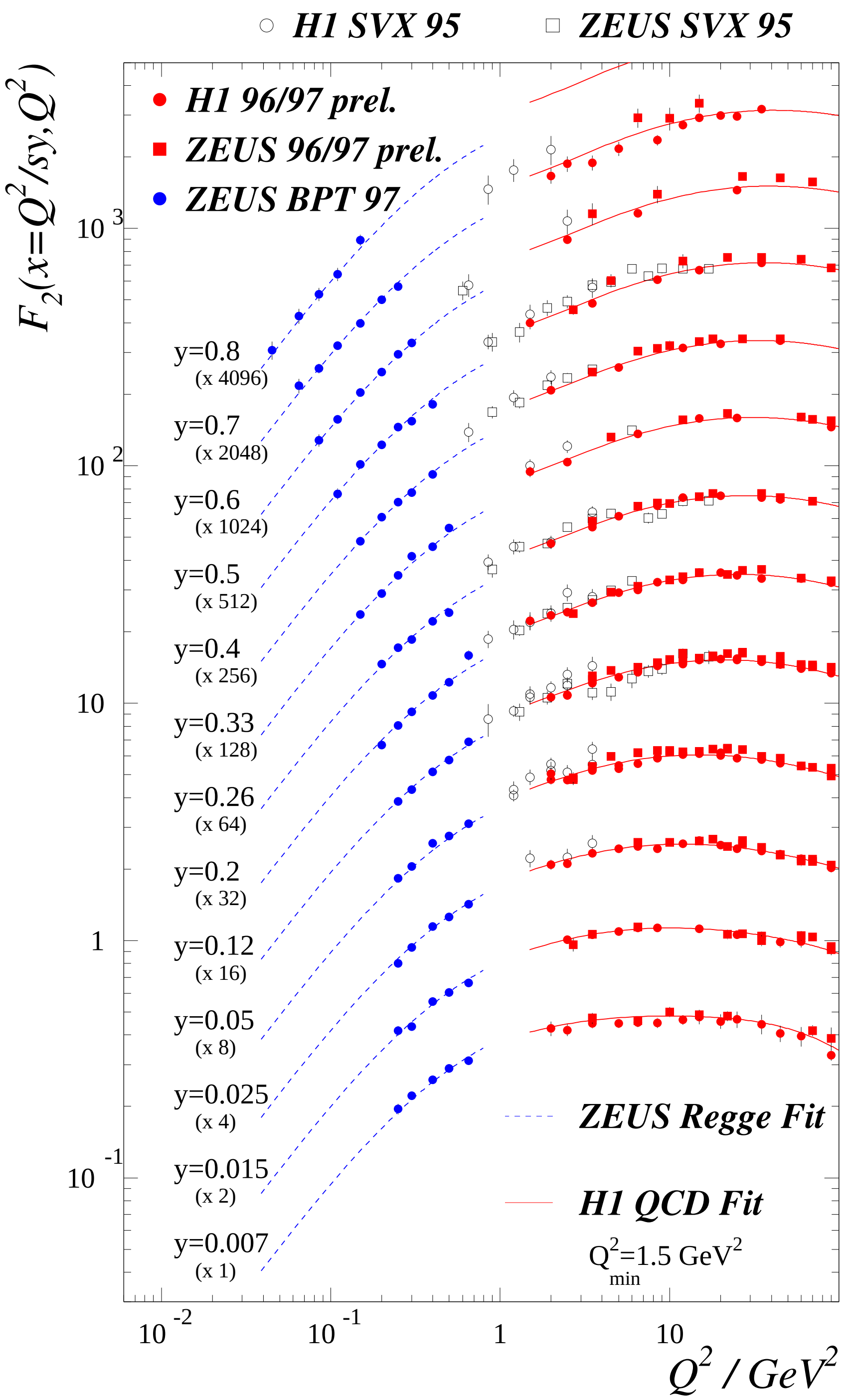
($\alpha_{\mathbb{P}} \approx$ 1.08, the "soft Pomeron") i.e. $W^{0.16}$



F_2 at $Q^2 < 1$ GeV
$10^{-3} > x > 10^{-6}$

Recall $y = W^2/s$

F_2 vanishes like Q^2 at fixed W.



Regge fit:

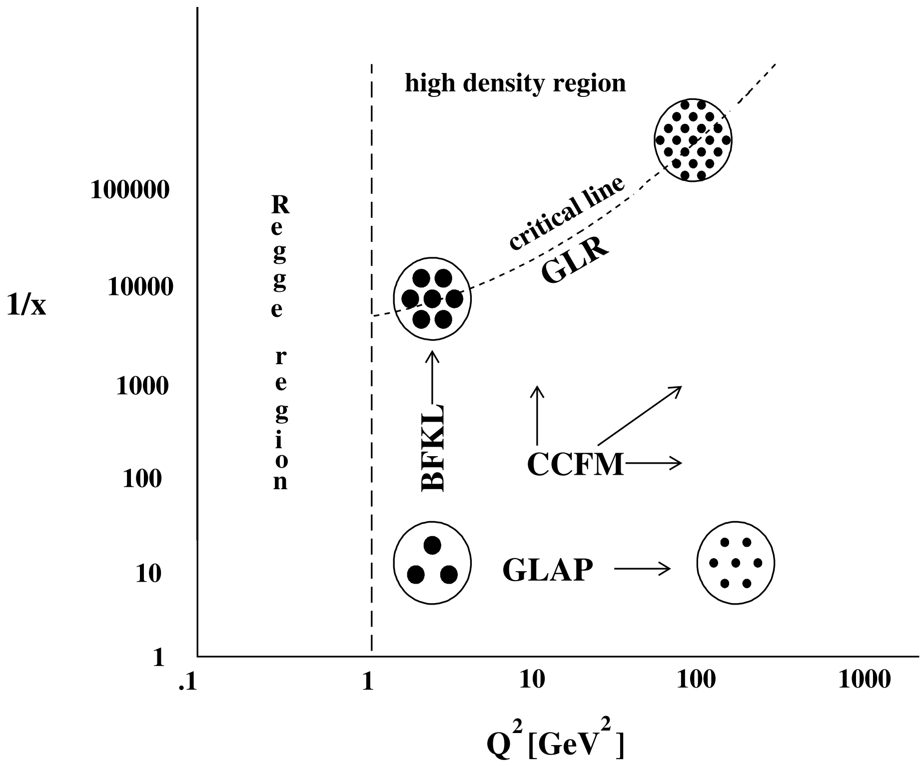
$$F_2(x,Q^2) = \left(\frac{Q^2}{4\pi^2\alpha}\right) \cdot \left(\frac{M_0^2}{M_0^2+Q^2}\right) \cdot$$
$$\left(A_{\mathbb{R}} \cdot \left(W^2\right)^{\alpha_{\mathbb{R}}-1} + A_{\mathbb{P}} \cdot \left(W^2\right)^{\alpha_{\mathbb{P}}-1}\right)$$

$\alpha_{\mathbb{P}} \approx 1.1$ and $M_0 \approx 0.7$ GeV.

Summarizing thus far..

- GLAP fits ($\ln Q^2$ resummation of QCD) can describe data well above $Q^2 > 1$ GeV^2
- Regge fits can describe data well $Q^2 < 1$ GeV^2

BUT...



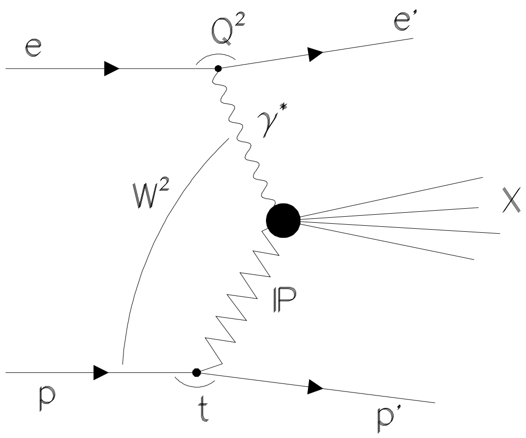
- What about $\ln 1/x$ (BFKL,CCFM), higher twist, terms etc.?
- What about shadowing or saturation (GLR): i.e. effects from high density of partons at low-x?

Let's look at DIS diffraction $\Rightarrow$

II. Diffraction in DIS

INCLUSIVE DIFFRACTION

At HERA diffractive dissociation of virtual photons is observed ($\approx$ 10%): can we understand this in terms of pQCD?



Exchange of a colorless object. (Pomeron)

Define 3 more variables:

$x_{\mathbb{P}}$: momentum fraction taken by the pomeron.

β: momentum fraction of the struck parton in the pomeron.

t: momentum transfer at the proton vertex.

- Beyond the simple GLAP picture : Need to interact with 2 gluons, at least!
- What is the relationship of diffraction with F_2 at small-x?

The diffractive cross section is written as:

$$\frac{d^3\sigma^D}{d\beta dQ^2 dx_{\mathbb{P}}} = \frac{4\pi\alpha^2}{\beta Q^2}(1 - y + \frac{y^2}{2})F_2^{D(3)}(\beta, Q^2, x_{\mathbb{P}})$$

(integrated over t)

Hard factorization has been proven (Collins 1997):

$$F_2^D \sim f^D \otimes \hat{\sigma}$$

Where f^D is the diffractive parton densities.

If Regge factorization is assumed:

$$F_2^D(x_{\mathbb{P}}, t, Q^2, \beta) = f(x_{\mathbb{P}}, t) \cdot F_2^{\mathbb{P}}(\beta, Q^2)$$

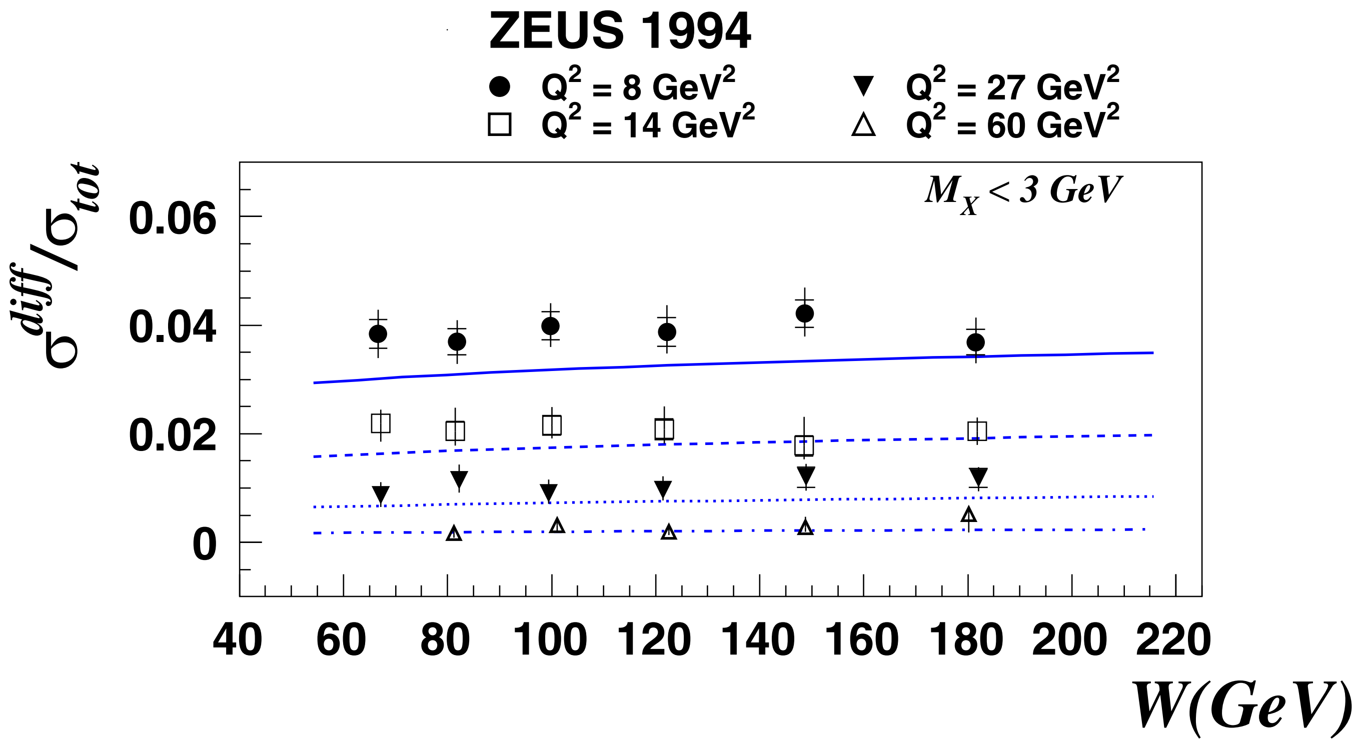
$\Rightarrow$ Can do a GLAP analysis of $F_2^{\mathbb{P}}$, the universal (for DIS) Pomeron structure function. (More from Max..)

$f(x_{\mathbb{P}}, t) \approx 1/x_{\mathbb{P}}$, the Pomeron flux factor.

However....

$\Rightarrow$ The origin of the Pomeron flux unexplained.

$\Rightarrow$ What is the relationship of σ_{diff} to σ_{tot}?



- Diffraction has the same W^2 (or x) dependence as the total cross section! ($\sigma_{diff} \sim W^{0.4}$, $Q^2 \sim 10$ GeV^2)

- Contradicts naive optical theorem expectation $\Rightarrow \sigma^{tot}_{\gamma^* p} \sim W^a$ then $\sigma^{diff} \sim W^{2a}$
- Contradicts expectation–if $\sigma^{tot}_{\gamma^* p} \sim$ Gluon, then $\sigma^{diff} \sim$ Gluon^2 ??

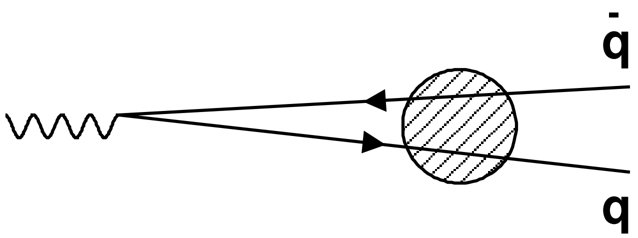
- $\Rightarrow$ Also contradicts expectation of Regge: $\alpha_{\mathbb{P}} = 1.08$ (i.e. $\sim W^{0.16}$)

The lines in the figure are from the saturation model of Golec-Biernat and Wüsthoff

IMPACT PARAMETER SPACE (OR DIPOLE) PICTURE

(over-)simplified

- Proton dissociates to a $q\bar{q}$ pair (dipole) upstream of the protons



- The interaction is written in terms of the dipole cross section $\hat{\sigma}_{dipole}(r\ldots)$ The primary dependence is on r, the radius of the dipole
- Q^2 also enters via the photon wave function ($\Psi_\gamma \Rightarrow$ determines the radius r)

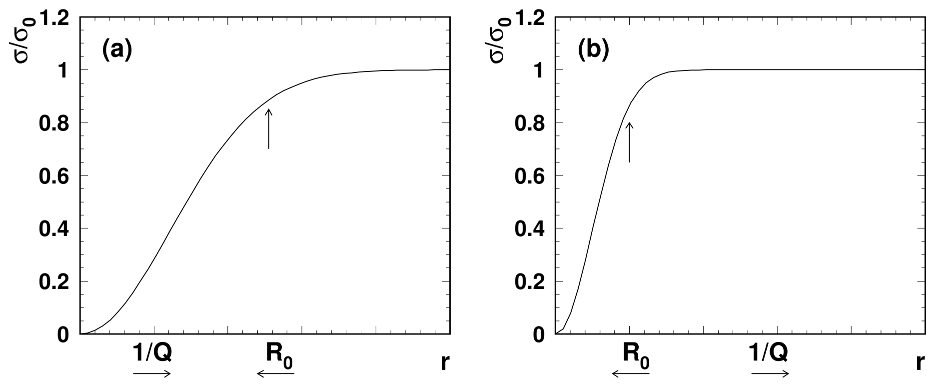
$$\sigma^{diff} \sim \int d^2r \int dz |\Psi(z,r)|^2 \hat{\sigma}^2_{dipole}(r..)$$

z: momentum sharing of the dipole.

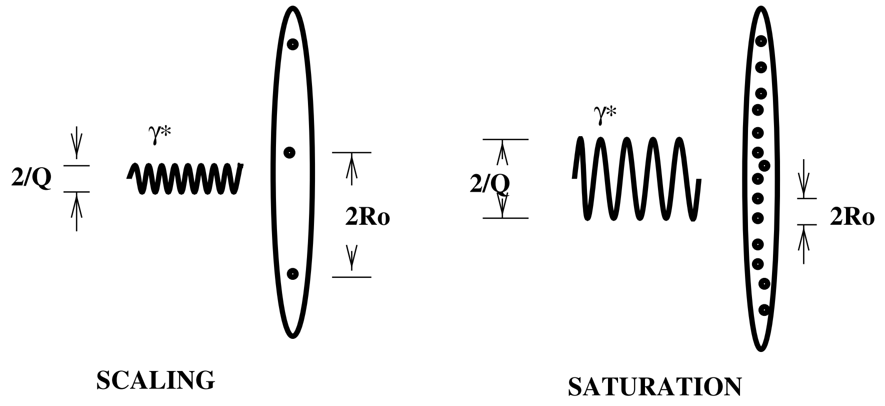
$\Rightarrow$ $\hat{\sigma}_{dipole}$ is also applicable to σ_{tot} (i.e. F_2) and VM production!

The model by Golec-Biernat and Wüsthoff (GB&W)

- $\hat{\sigma}_{dipole}$ increases as r^2 at small r. $\Rightarrow$ Color transparency.
- $\hat{\sigma}_{dipole}$ becomes constant (saturates) at large r.

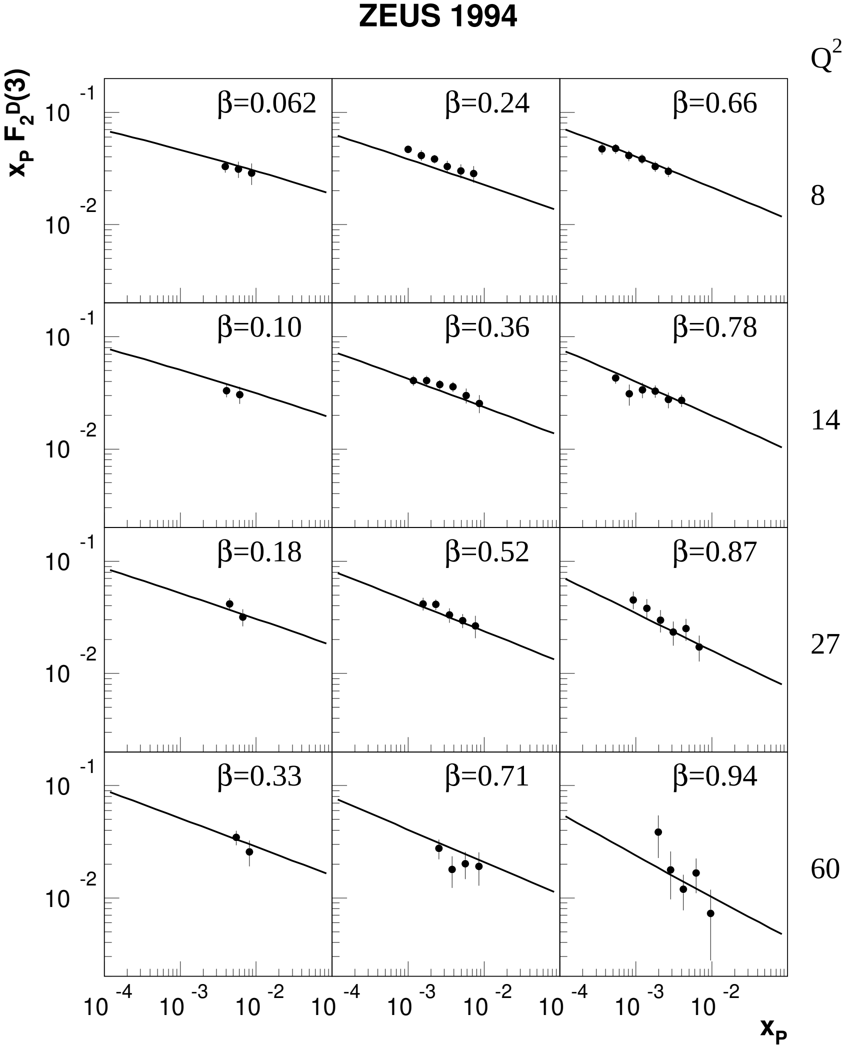


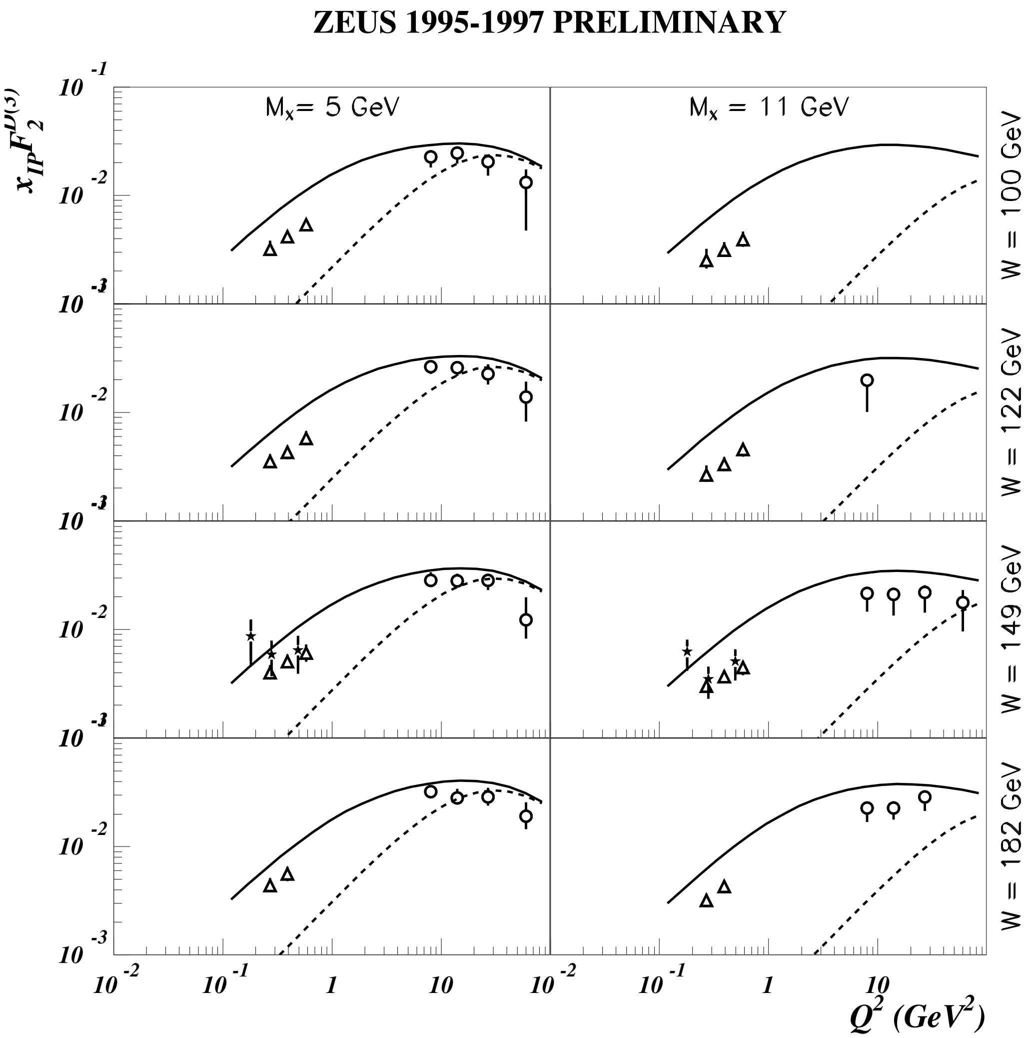
- At what r the cross section saturates depends on the density of the partons, i.e. x. $\rightarrow \hat{\sigma}_{dipole}(x, r)$ [R$_0 \sim 1/gluon \sim x^{\lambda}$]



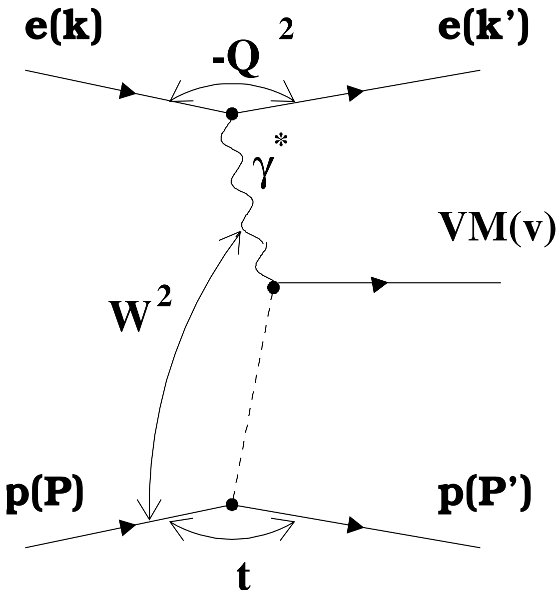
- The W (or x) dependence of σ_{diff} is moderated by saturation. $\rightarrow$ lines in the ratio plot.

Description of Diffractive F_2 in the GB&W Model





DIFFRACTIVE VECTOR MESON PRODUCTION



- Very similar process to the inclusive diffraction
- What is the W dependence?

  Recall:

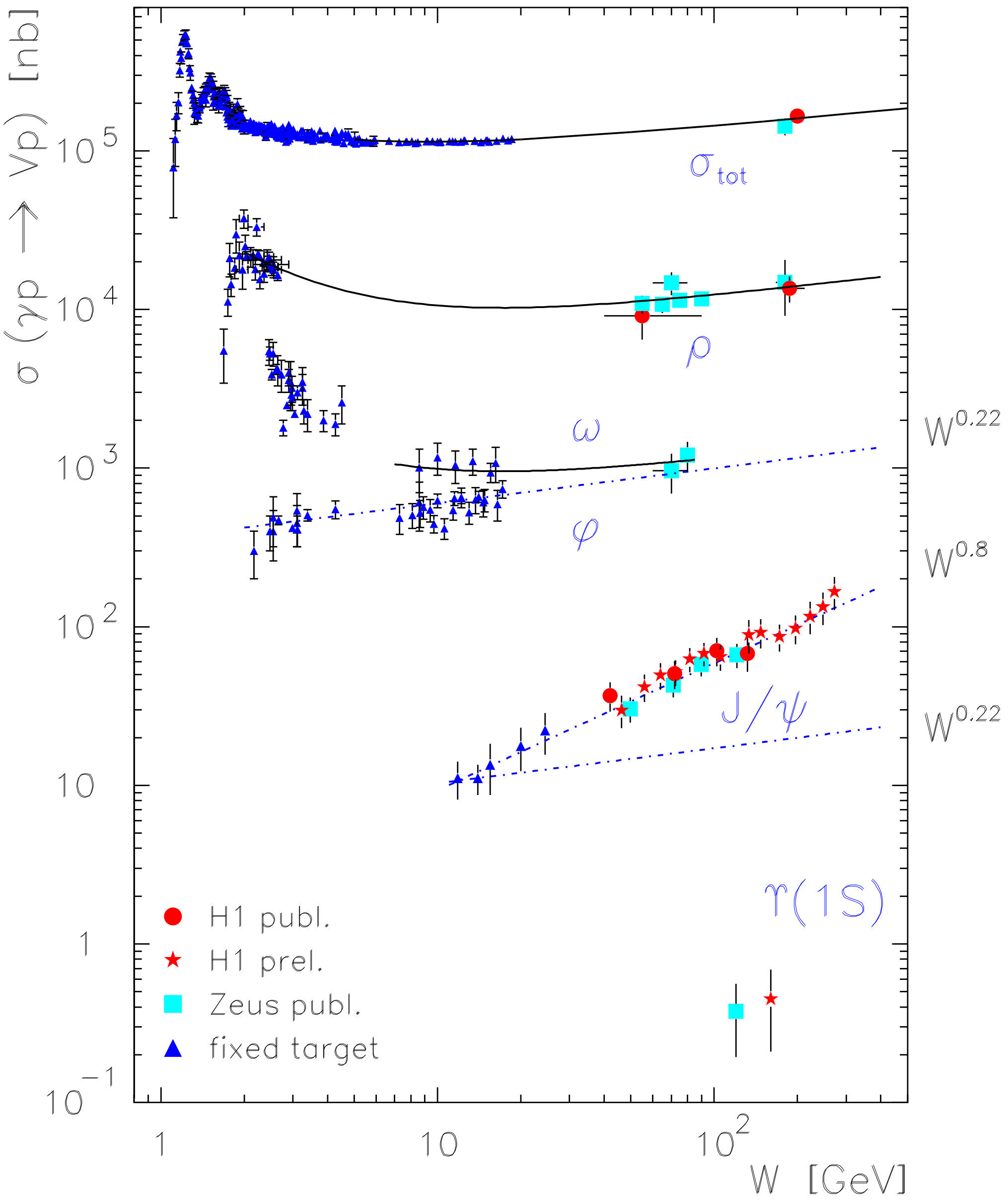
  $\sigma_{\gamma p}^{tot} \sim W^{2(\alpha_{I\!P}-1)} \approx W^{2(0.08)} = W^{0.16}$

  F_2^p (or $\sigma_{\gamma^* p}$) (at $Q^2 \approx 10$ GeV^2) $\sim x^{-0.2}$

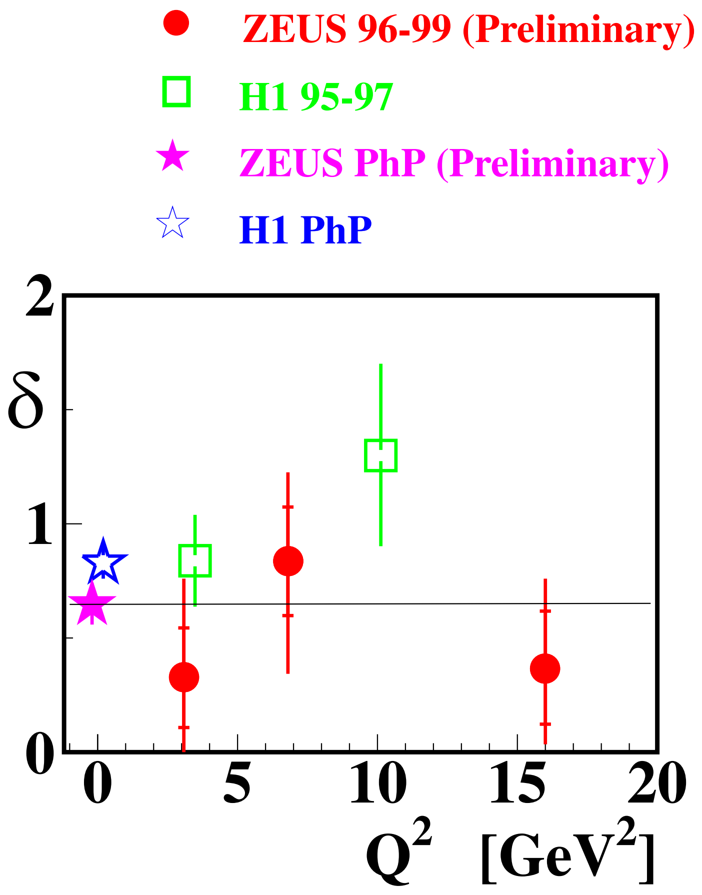
  $\Rightarrow W^{2(0.2)} = W^{0.4}$

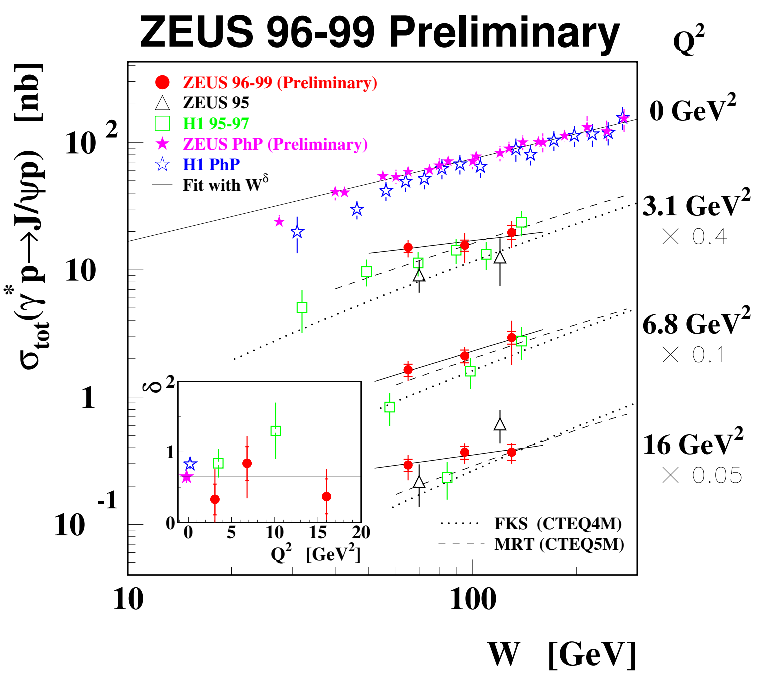
  $\sigma^{diff}/\sigma_{\gamma^* p}$ constant.
- What about Vector Meson production $\Rightarrow$

PHOTPRODUCTION OF VECTOR MESONS



J/ψ rises like $W^{0.8}$! (i.e. twice F_2 at 10 GeV^2)

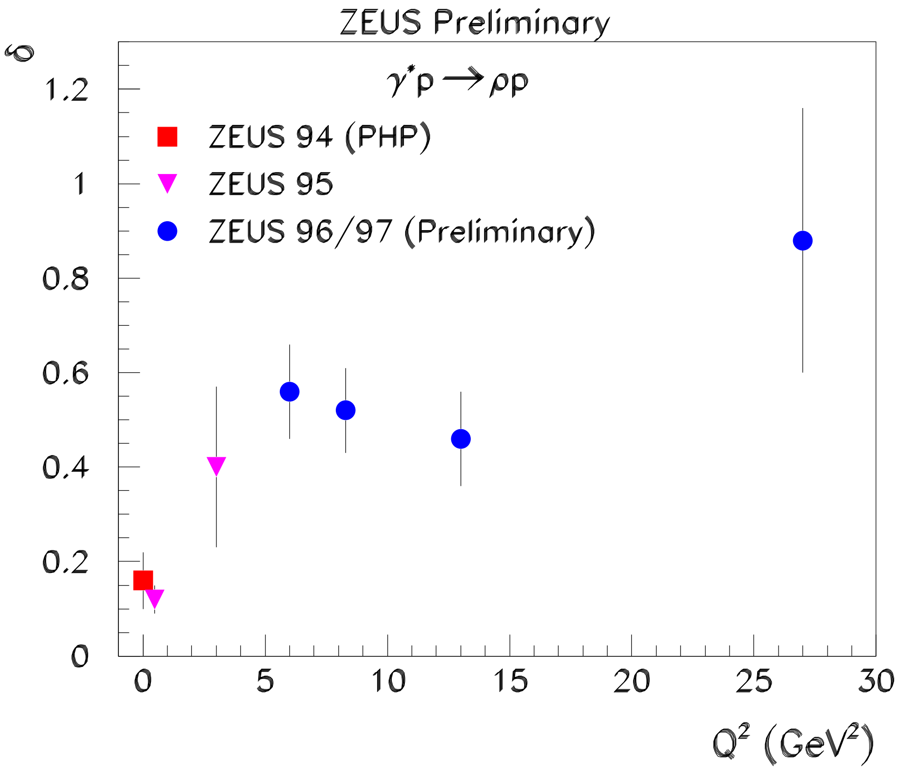




$\sigma \propto W^{\delta}$

Still large errors: slope (δ) consistent with being constant as function of Q^2.

What about the Q^2 dependence of ρ's? $\Rightarrow$

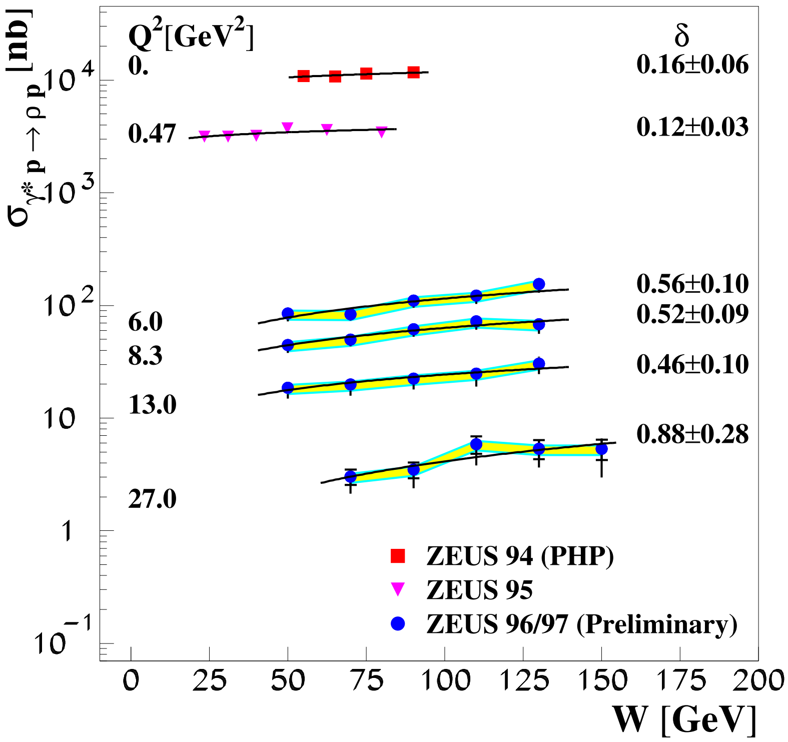


$\sigma \propto W^{\delta}$

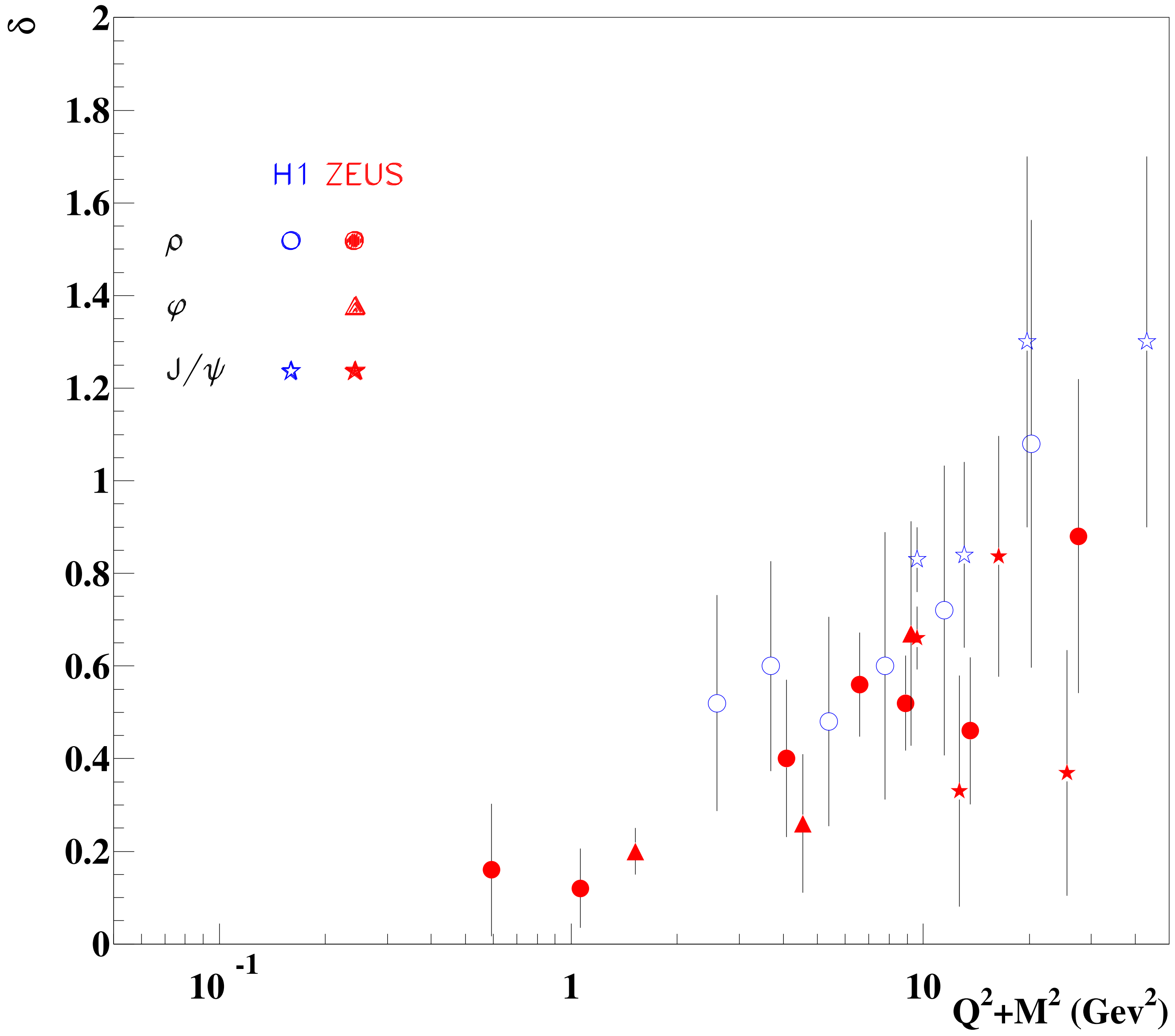
δ increases with Q^2

At Q^2 of 27 GeV^2,

$\sigma \sim W^{0.88}$!



Elastic VM Production at HERA



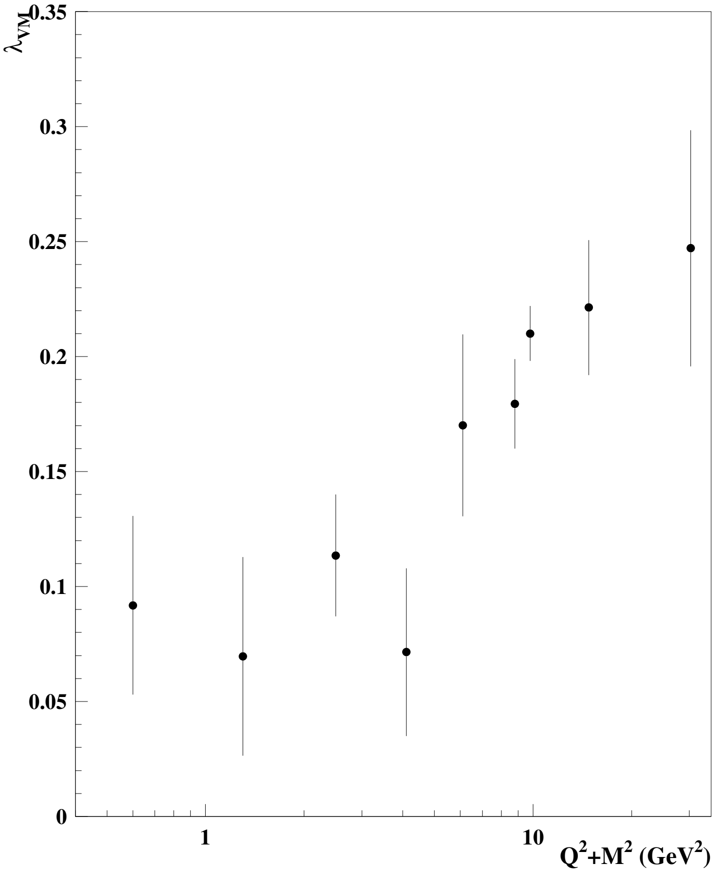
The rise, δ, scales with $Q^2 + M^2_{VM}$!

Defining λ_{VM} (where $\sigma \sim x^{-2\lambda_{VM}}$)

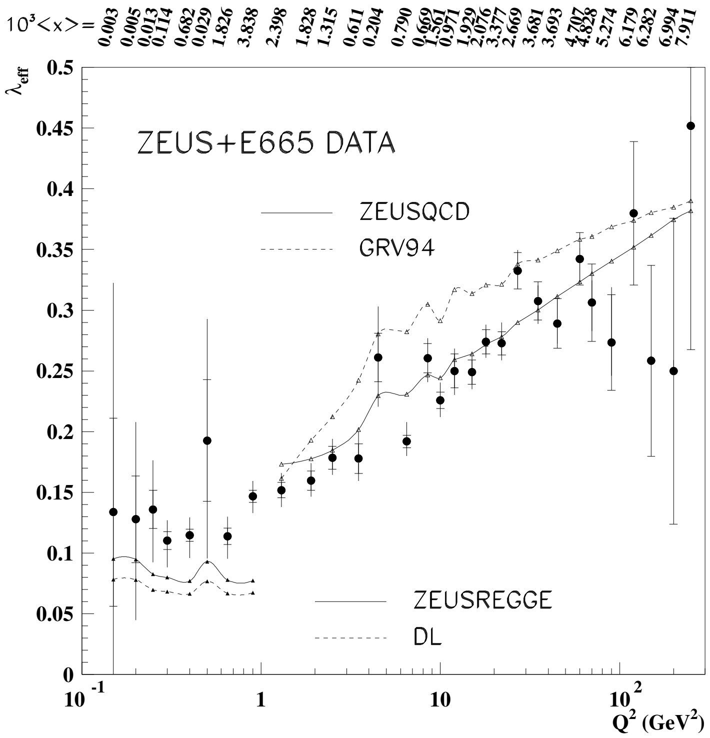
Recall: $F_2^p \sim x^{-\lambda}$ so there's a factor 2

$2\lambda = \delta = 4\lambda_{VM}$! (apologies..)

Elastic VM Production at HERA



Inclusive DIS



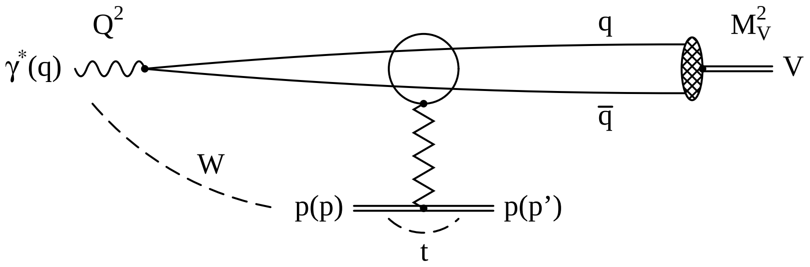
λ_{VM} (at $Q^2 + M^2_{VM}$) $\approx \lambda_{F_2}$ (at Q^2)

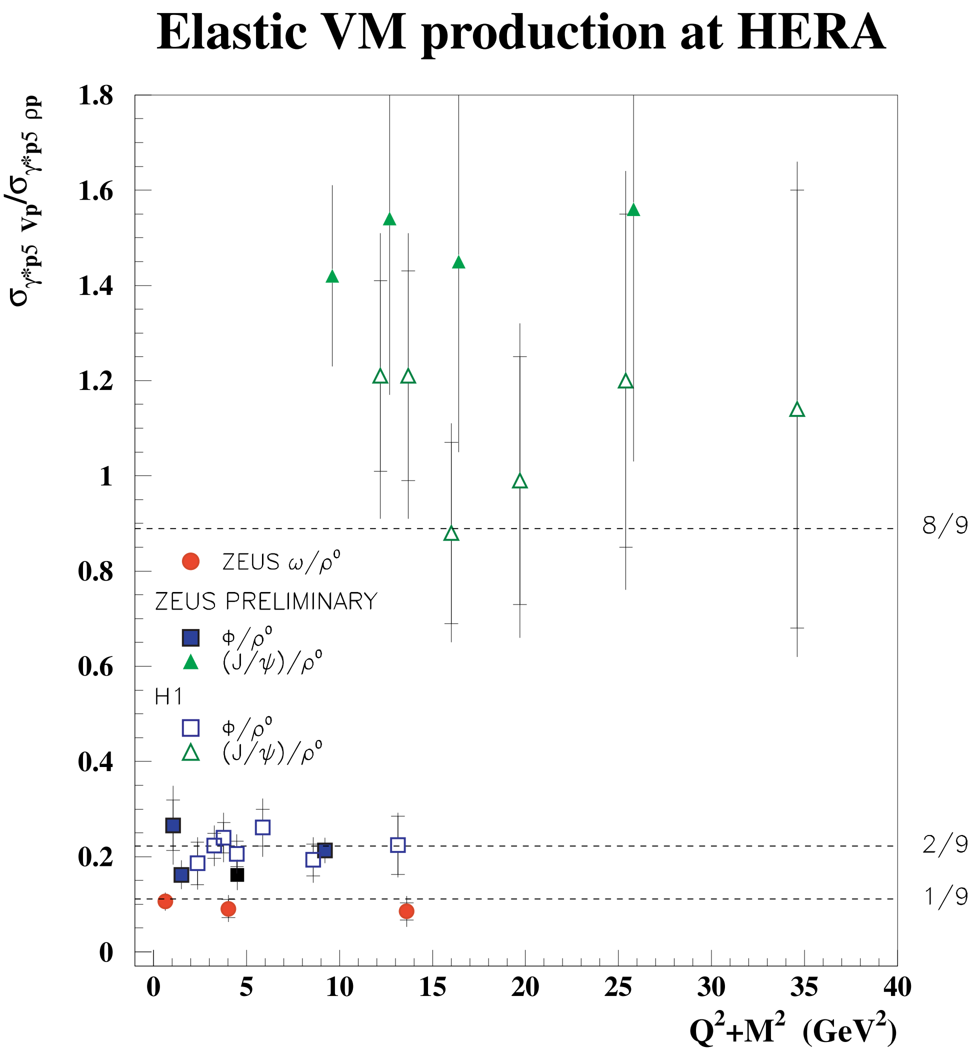
$\Rightarrow$ VM rises approximately twice as fast as F_2.

Look at the ratios of different species of VM $\Rightarrow$

Naively: $\sigma_{VM} \sim f_V^2 \cdot \hat{\sigma}$?

i.e. coupling times the (dipole?) cross section $\hat{\sigma}$?

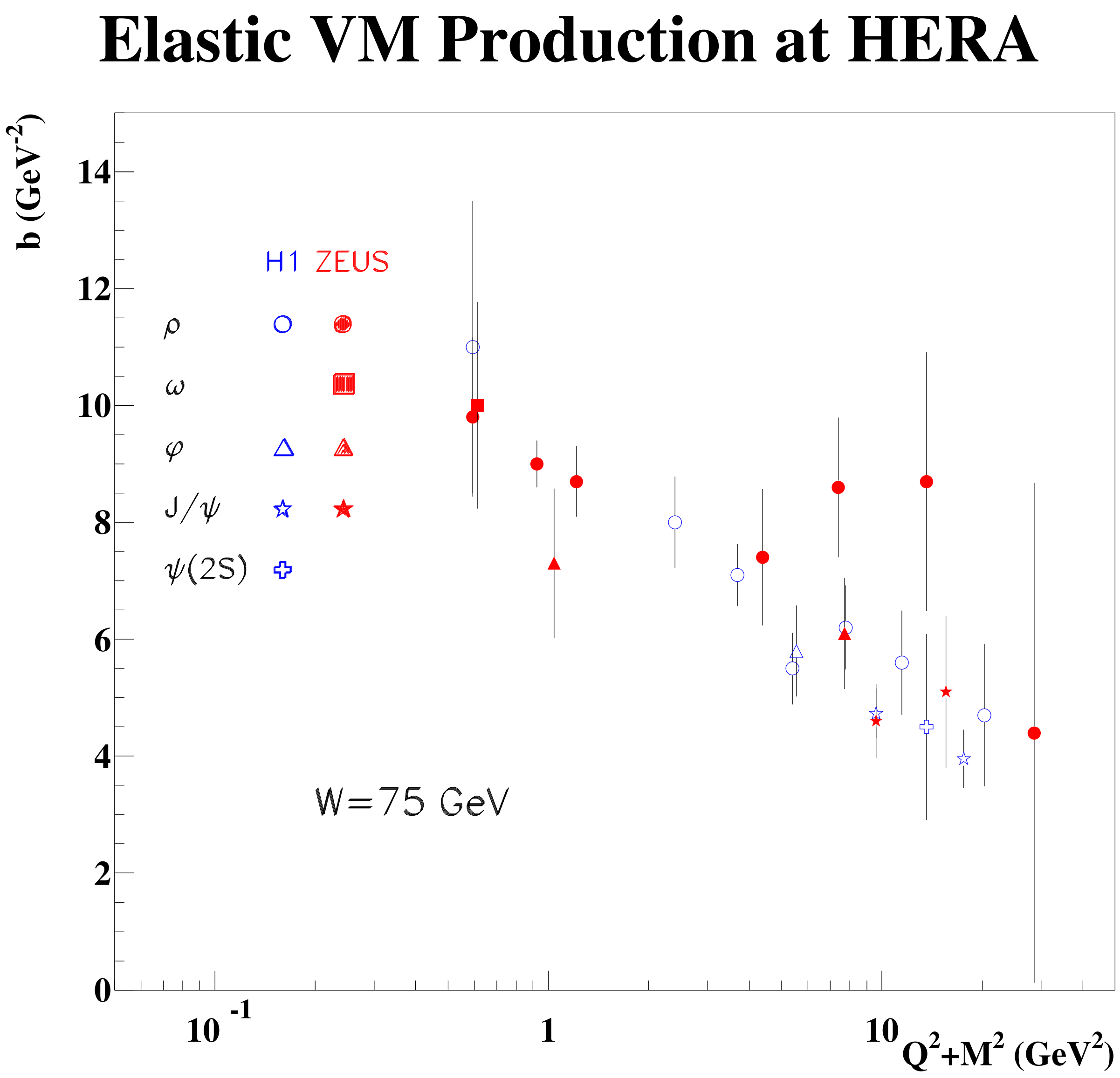




f_V^2 from charges of the quarks: ρ:ω:ϕ:J/ψ $\rightarrow$ 9:1:2:8.

... and $\hat{\sigma}$ that depends on the dipole radius?

Look at the t slope: $\frac{d\sigma_{VM}}{d|t|} \propto e^{-b|t|}$



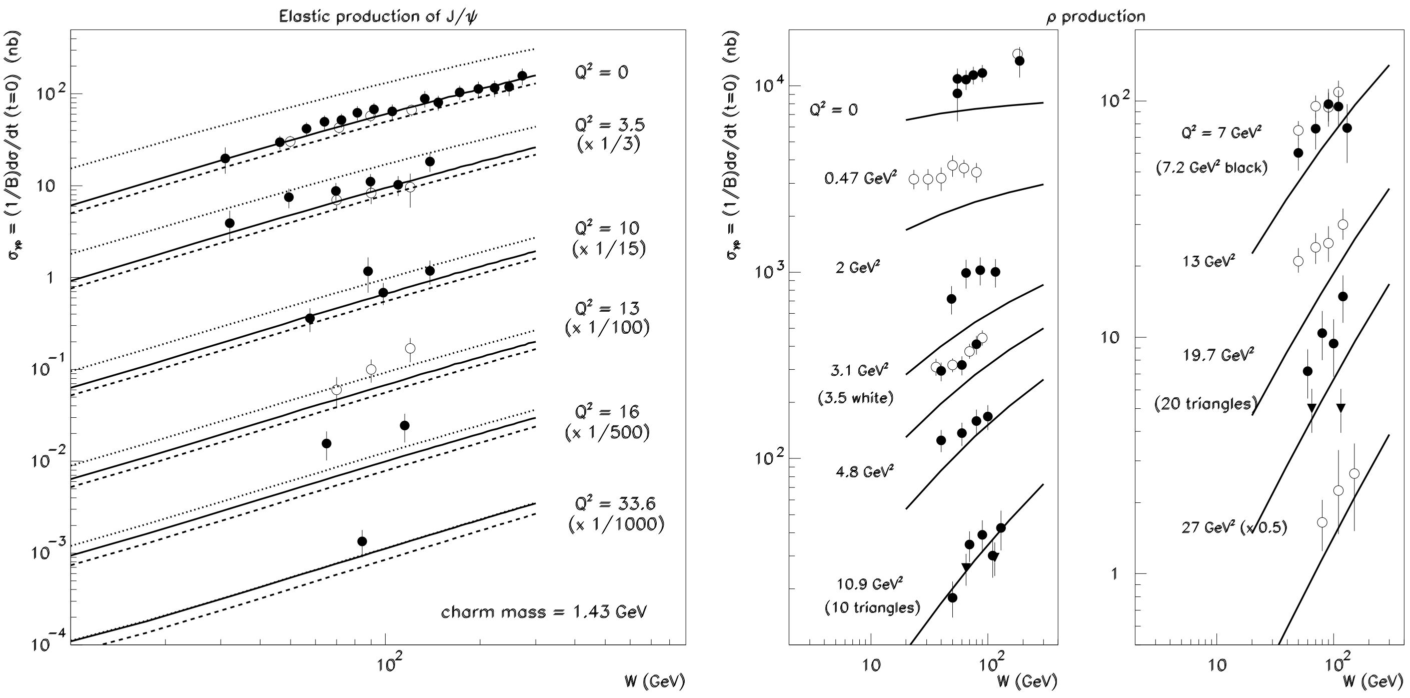
Consistent with $Q^2 + M^2_{VM}$ giving the "size" of the interaction.

Dipole formulation of the VM cross section $\Rightarrow$

Amplitude: $A = \int dz \int d^2 r \Psi_{VM} \hat{\sigma}_{dipole} \Psi_{\gamma}$

Ψ_{VM}: The vector meson wave function.

Ψ_{γ}: The photon wave function.

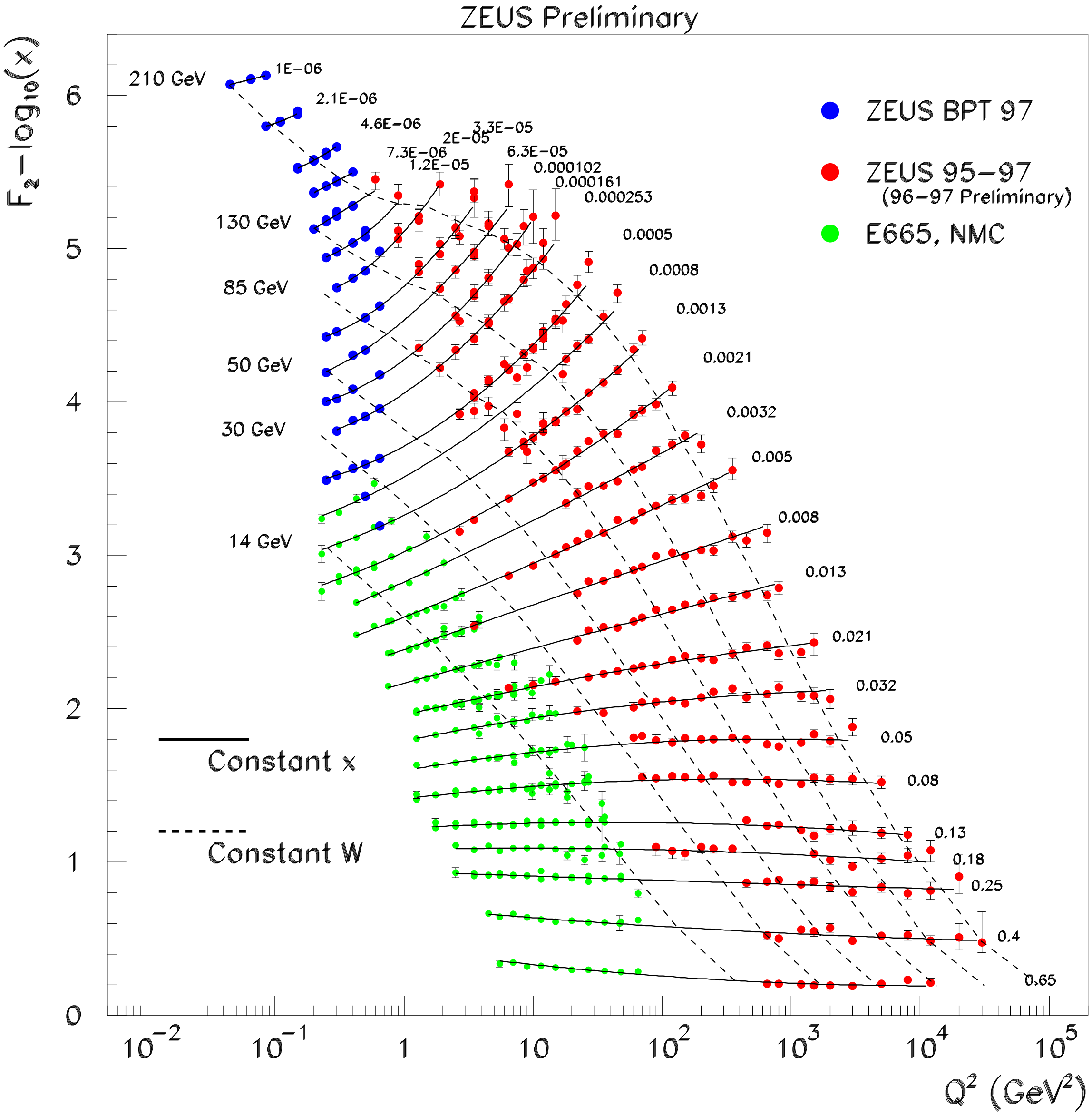


Calculations from G-B&W model by M. Soares.

(More details later from H. Kowalski)

III. F_2 at Small-x Revisited

Do we really understand all of this?



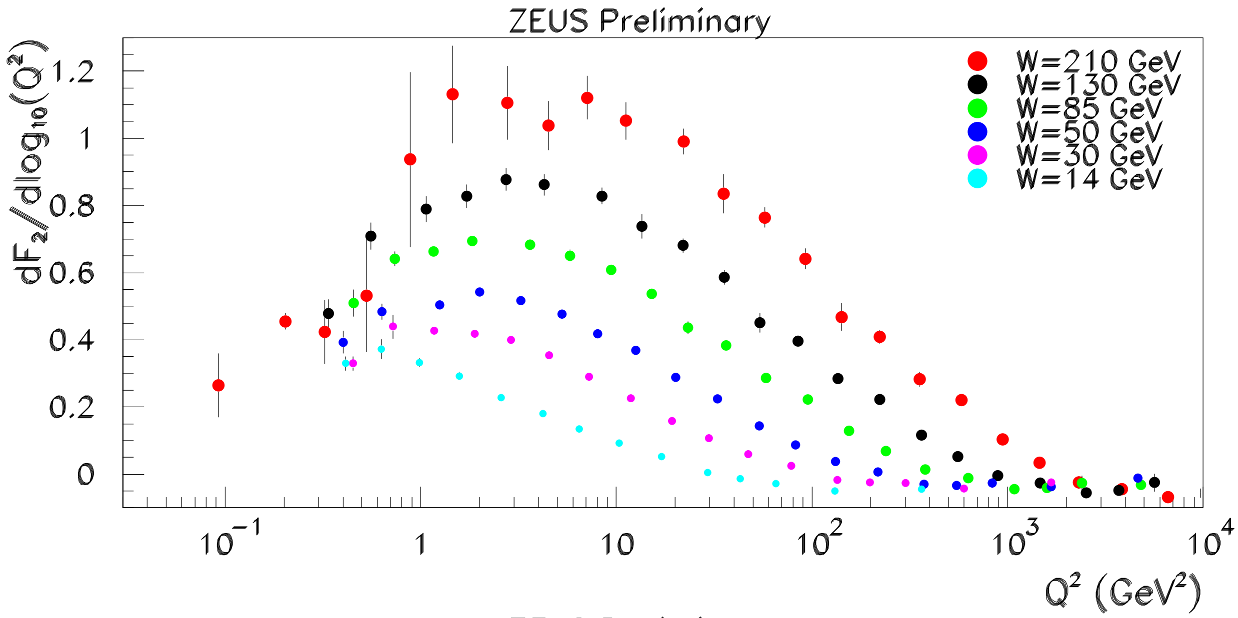
What is happening at low x and medium Q^2?

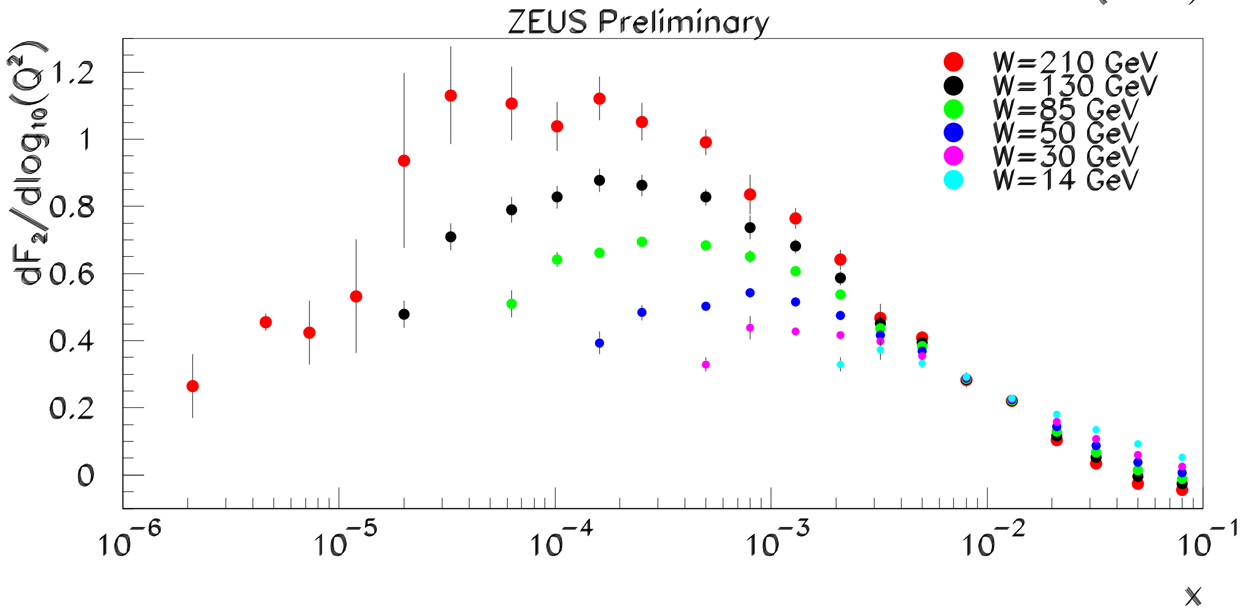
note: offset constant is $\log x$ i.e. no distortion

Lines are fits $A(x) + B(x)logQ^2 + C(x)(logQ^2)^2$

How to make the transition more visible:

$\frac{dF_2}{dlogQ^2}\ |_x = B + 2C(logQ^2)$ at fixed W

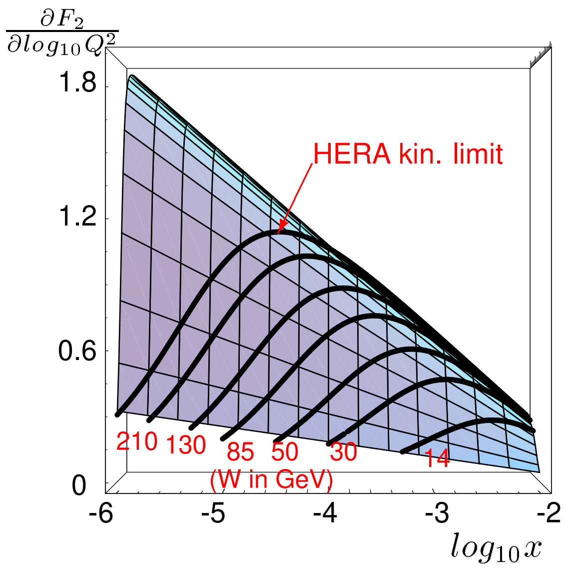


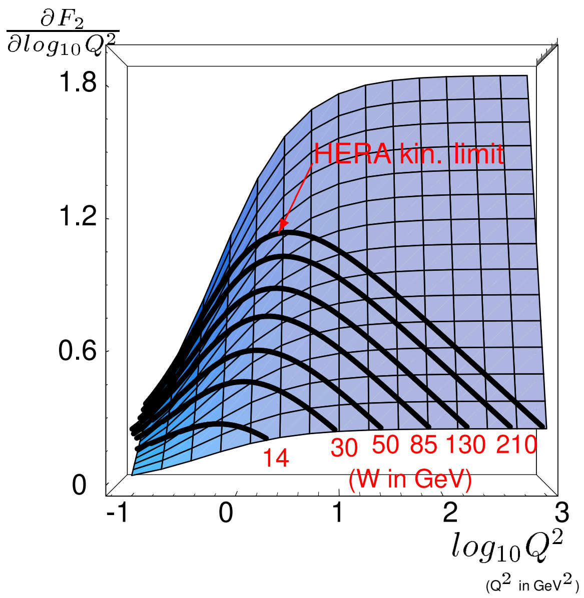


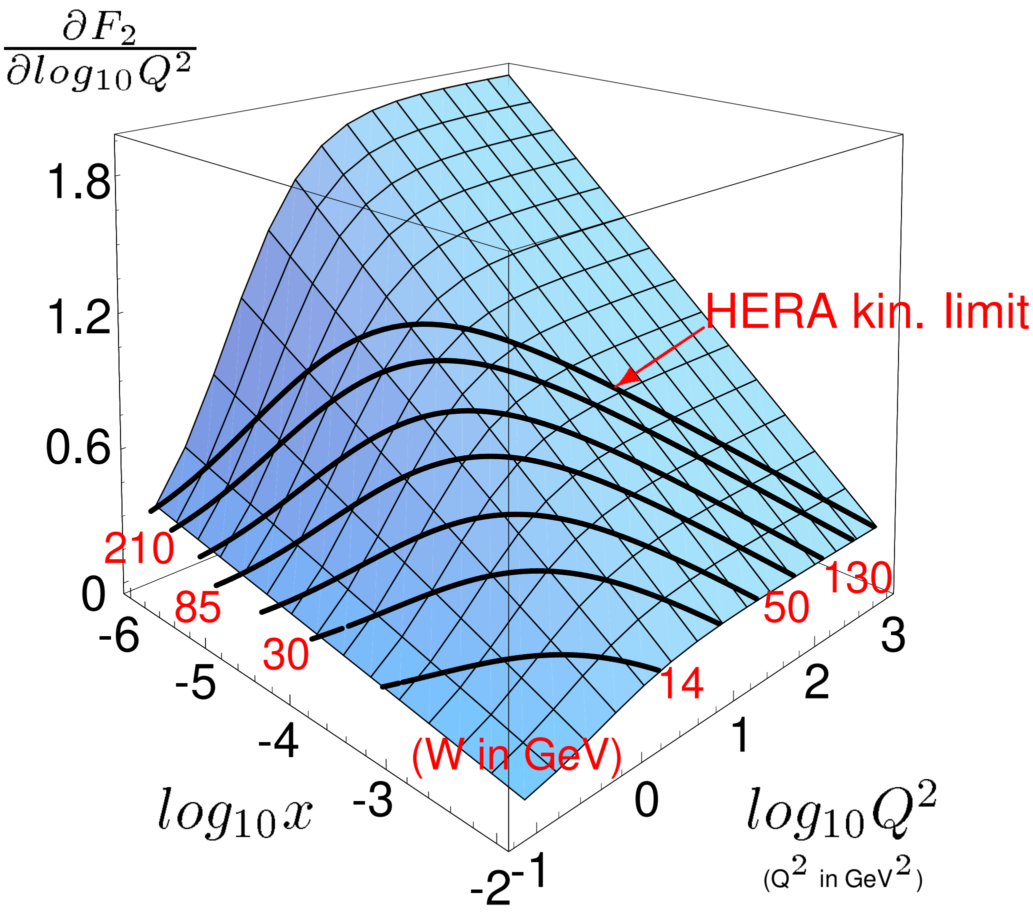
What does this shape mean? $\Rightarrow$

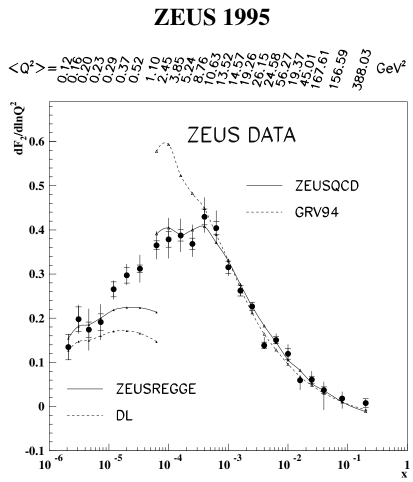
(Remember $W^2 = Q^2/x$ at low-x)

Qualitative Features of $\frac{dF_2}{dlogQ^2}$ in 2 Dimensions





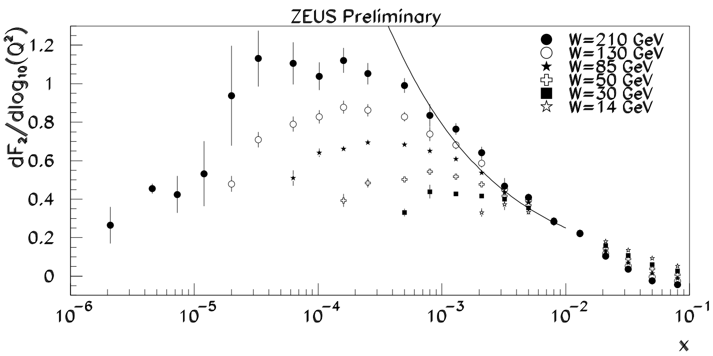




Naively, in the region where GLAP applicable (Q^2 large):

$$dF_2/dlog(Q^2) \sim xg \sim x^{-\lambda}$$

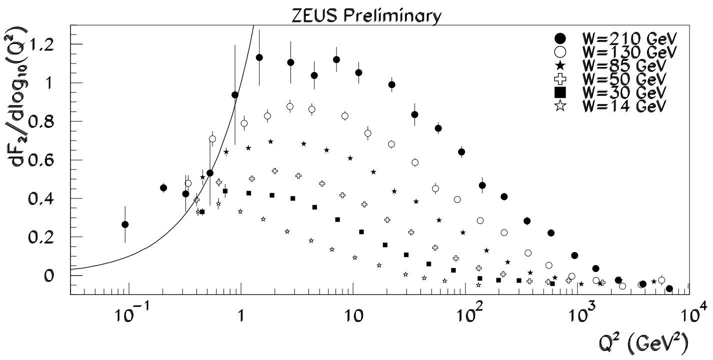
IF λ is a slow function of x and Q^2.
$\Rightarrow Q^2$ is not an important scale. x is the important variable in the right hand side of the plots.



In the low Q^2 region: $F_2 \sim Q^2\sigma_0$, i.e. vanishing like Q^2. Therefore:

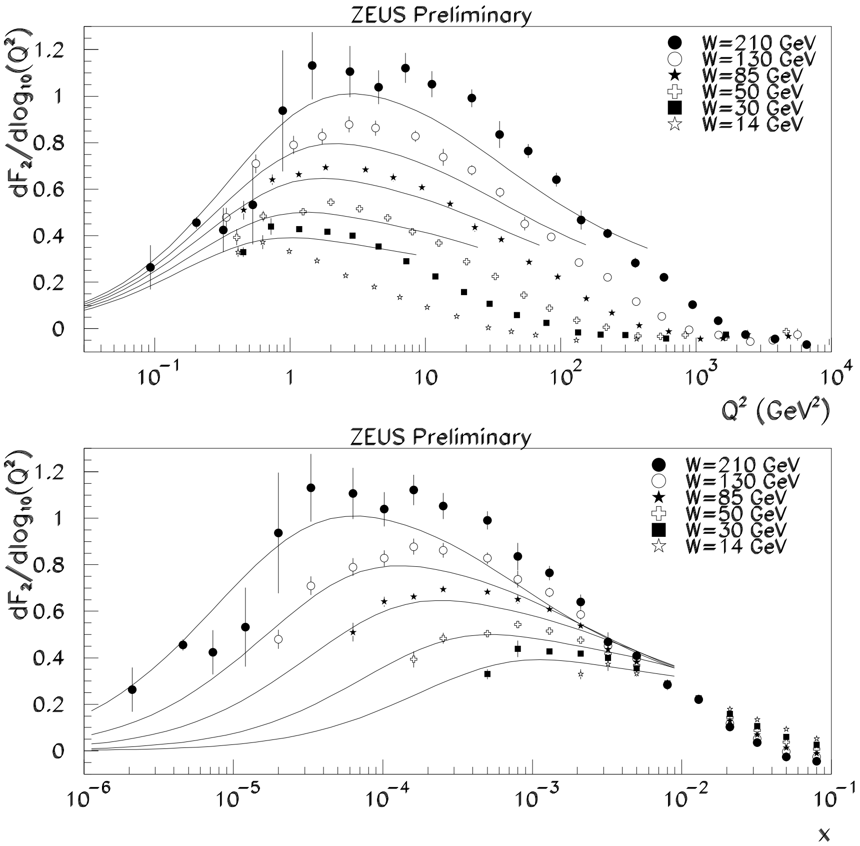
$$dF_2/dlog(Q^2) \sim Q^2\sigma_0$$

$\Rightarrow x$ is not an important variable. Q^2 is the important variable in the left hand side of the plots.



DIPOLE formulation of DIS

$$\sigma = \int d^2r \int dz |\Psi(z,r)|^2 \hat{\sigma}_{dipole}(x,r)$$

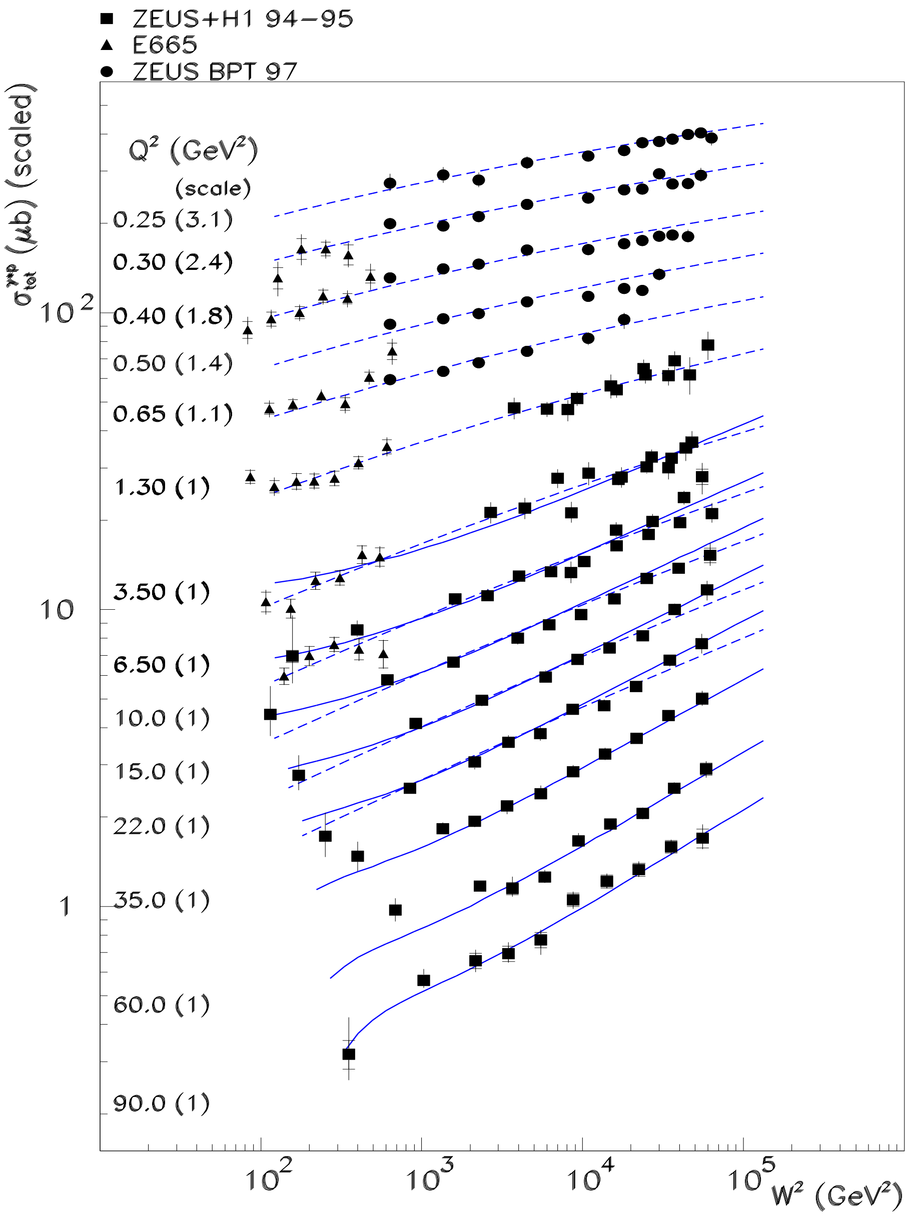


Using the same GB&W $\hat{\sigma}_{dipole}$ as for diffraction and VM. Has the correct qualitative behavior.

$x^{-\lambda}$ to Q^2 dependence arises from the dipole saturation mechanism. (i.e. cross section goes flat $\Rightarrow F_2$ falls like Q^2). Recall

$$\sigma_{tot}^{\gamma^* p} = \frac{4\pi^2\alpha}{Q^2} F_2$$

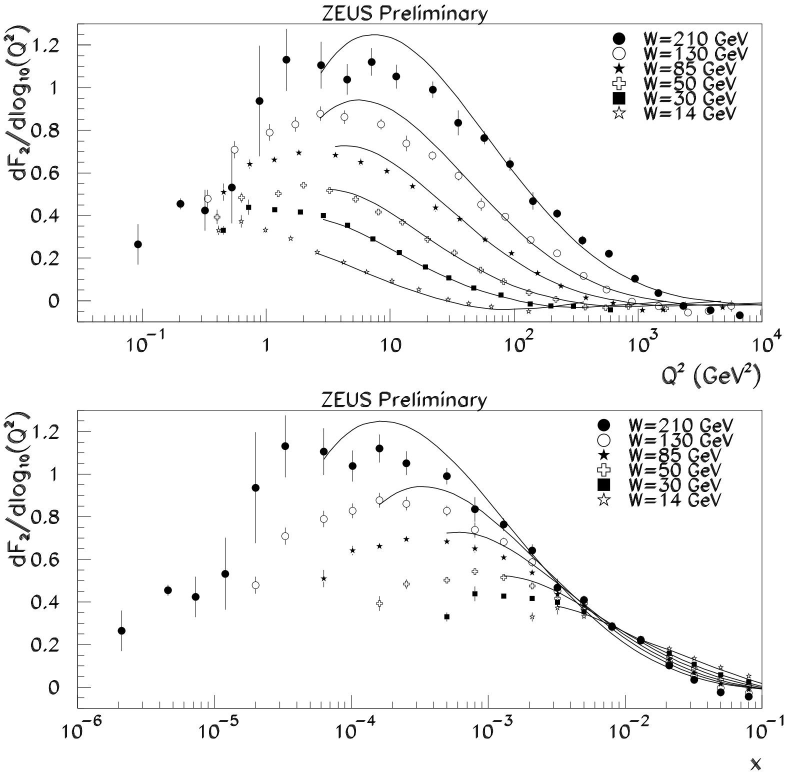
GB&W Description of $\sigma_{tot}^{\gamma^* p}$



Dotted line: GB&W model (3 parameter fit)

Solid line: MRST NLO QCD fit

$\Rightarrow$ What about $dF_2/dlog(Q^2)$ of NLO DGLAP fits....

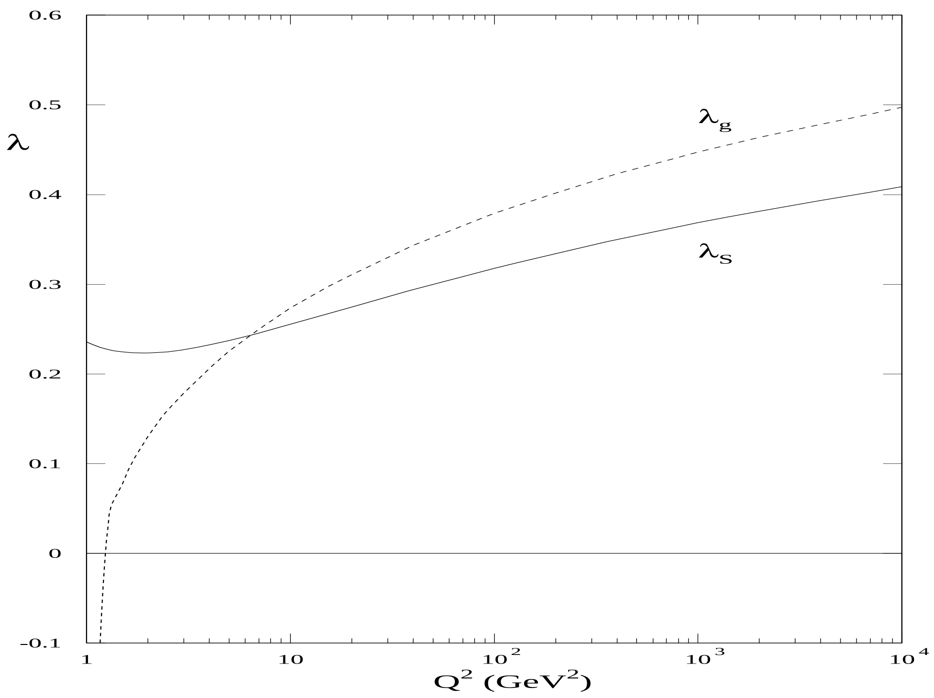


Lines: ZEUS NLO DGLAP fit.

How do the GLAP go around the corner?

λ_{gluon} falls rapidly below Q^2 of 10 GeV^2

MRST fit parameters



COMMENTS

- The appeal of the dipole approach is:

$\sigma_{tot} \sim \int d^2r \int dz |\Psi_\gamma|^2 \hat{\sigma}_{dipole}$
$\sigma_{diff} \sim \int d^2r \int dz |\Psi_\gamma|^2 \hat{\sigma}_{dipole}{}^2$
$\sigma_{VM} \sim |\int dz \int d^2r \Psi_{VM} \hat{\sigma}_{dipole} \Psi_\gamma|^2$

Data seems to indicates that this (qualitatively) works: i.e. $\hat{\sigma}_{dipole}$ measured from σ_{tot} gives the correct features of σ_{diff} and σ_{VM}!

- The appeal of the GB&W saturation model of the dipole cross-section is its intuitive physical picture.

and QUESTIONS (to theory)

- What is the relationship between the saturation of the $\hat{\sigma}_{dipole}$, at large r, and parton saturation at low-x. Does one imply the other?
- Can the NLO DGLAP fit of Q^2 down to 1 GeV^2 co-exist with the dipole interpretation of F_2? Are these interpretations mutually exclusive?
- Does the assumed shape of the dipole cross-section in G&B-W model (and models like it) imply that HERA has reached a ”new QCD regime” at a moderate Q^2 and low x? If so what does this mean? is it beyond DGLAP (to any order)? BFKL? beyond all pQCD?
- In diffraction: what is the relationship between the hard diffractive factorization and the dipole model?
- What is the meaning of pomeron and pomeron structure within the dipole model?
- What is the relationship of the dipole model to Regge and Vector Meson Dominance models?