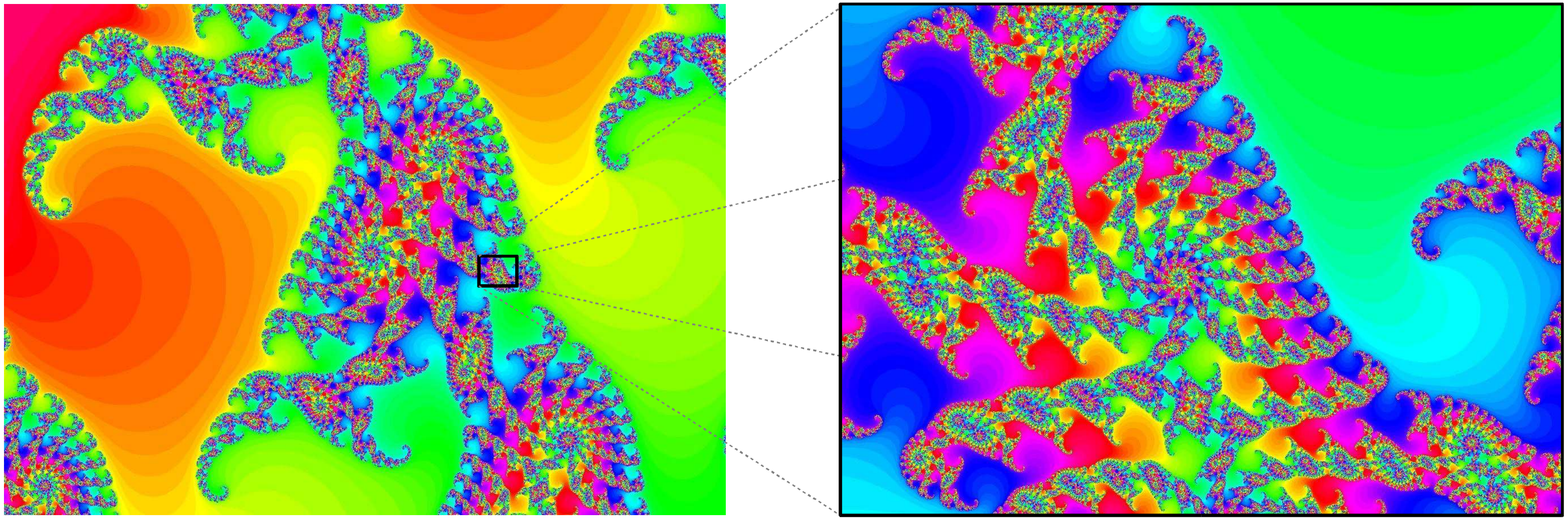
Self-similar Properties of the Proton Structure

Tomáš Laštovička (H1)
DESY Zeuthen, Charles University Prague
DIS 2002, Kraków, Poland

- **Self–similar Structures**
- **Fractal Dimension**
- **F_2 parametrisation**
- **Numerical Analysis – fit to F_2**
- **Summary**

Self-similar Structures?

- **structure is detailed and self−repeating at all scales**



- **Can this be applied to the proton structure?**

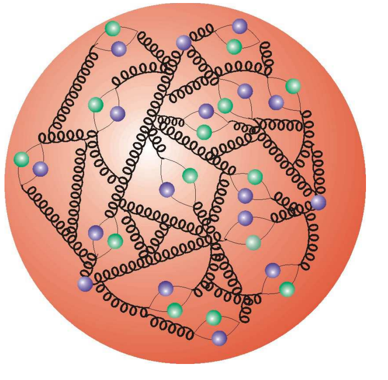
Fractals in Nature

Where fractal structures appear?

- Scale–independent structures
- Chaos
- Self–organized systems
- Cellular automats
  - Systems on lattice (e.g. 3D game ’life’, Lattice QCD(?))
- Signals
- ....

How to describe them?

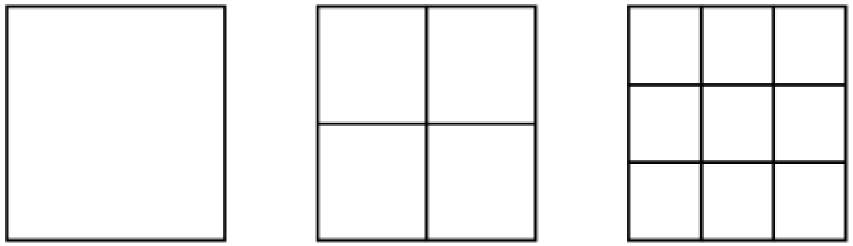
- Knowing the basic (iteration/evolution/...) rules of the system
- On Lagrangian/Hamiltonian level...
- Fractal Dimension
  - appropriate for the proton structure
  - straightforward check



Fractal Dimension

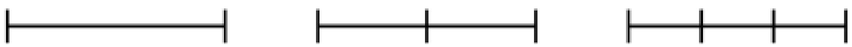
- **Redefinition of term 'dimension':**

2D



$$D = \frac{\log(No.\ selfsimilar\ pieces)}{\log(magnification\ factor)} = \frac{\log 4}{\log 2} = \frac{\log 9}{\log 3} = \mathbf{2}$$

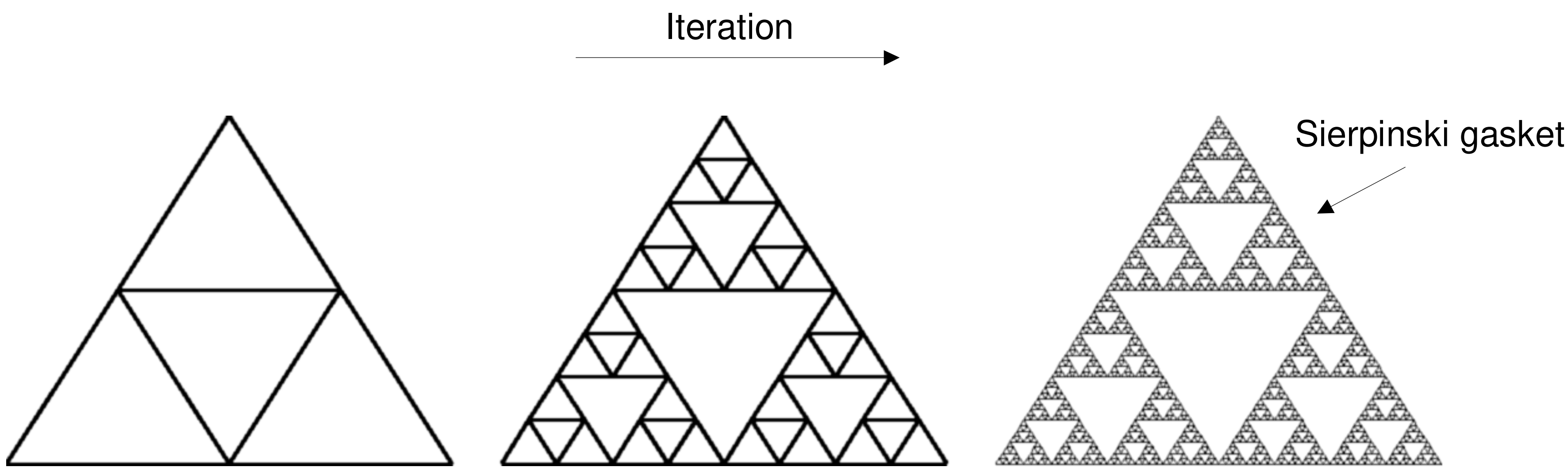
1D



$$D = \frac{\log 2}{\log 2} = \frac{\log 3}{\log 3} = \mathbf{1}$$

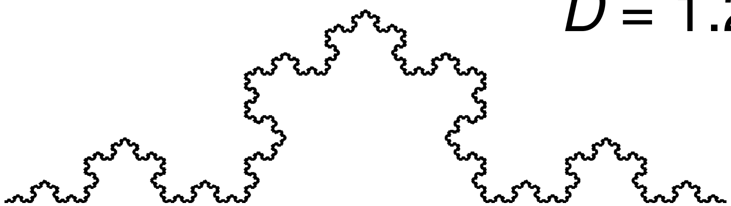
3D

Analogically for cube: $D = \frac{\log 8}{\log 2} = \frac{\log 27}{\log 3} = \mathbf{3}$

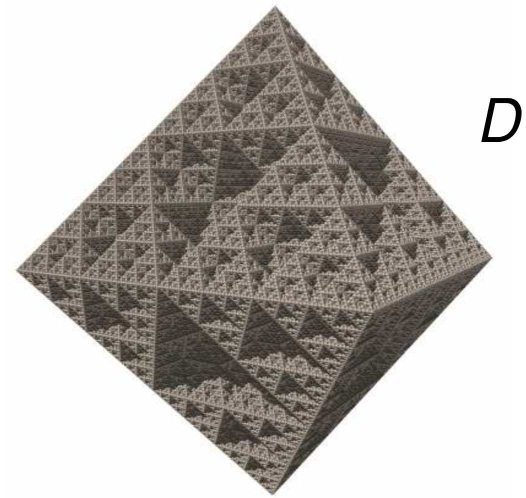


$$D = \frac{\log(No.\ selfsimilar\ pieces)}{\log(magnification\ factor)} = \frac{\log 3}{\log 2} = \frac{\log 27}{\log 8} \simeq \mathbf{1.585}$$

- **In general, fractals have *fractional dimension***



D = 1.262...



D = 2.585...

- **Generalization:**
  - **No. selfsimiliar pieces** **=> density function *f* (*z*)**
  - **Magnification factor** **=> real number *z***
- **Introducing *local dimension*:**

$$D(z) = \frac{\partial \log f(z)}{\partial \log z}$$

- **For a region where *D*(*z*) = *D* :**

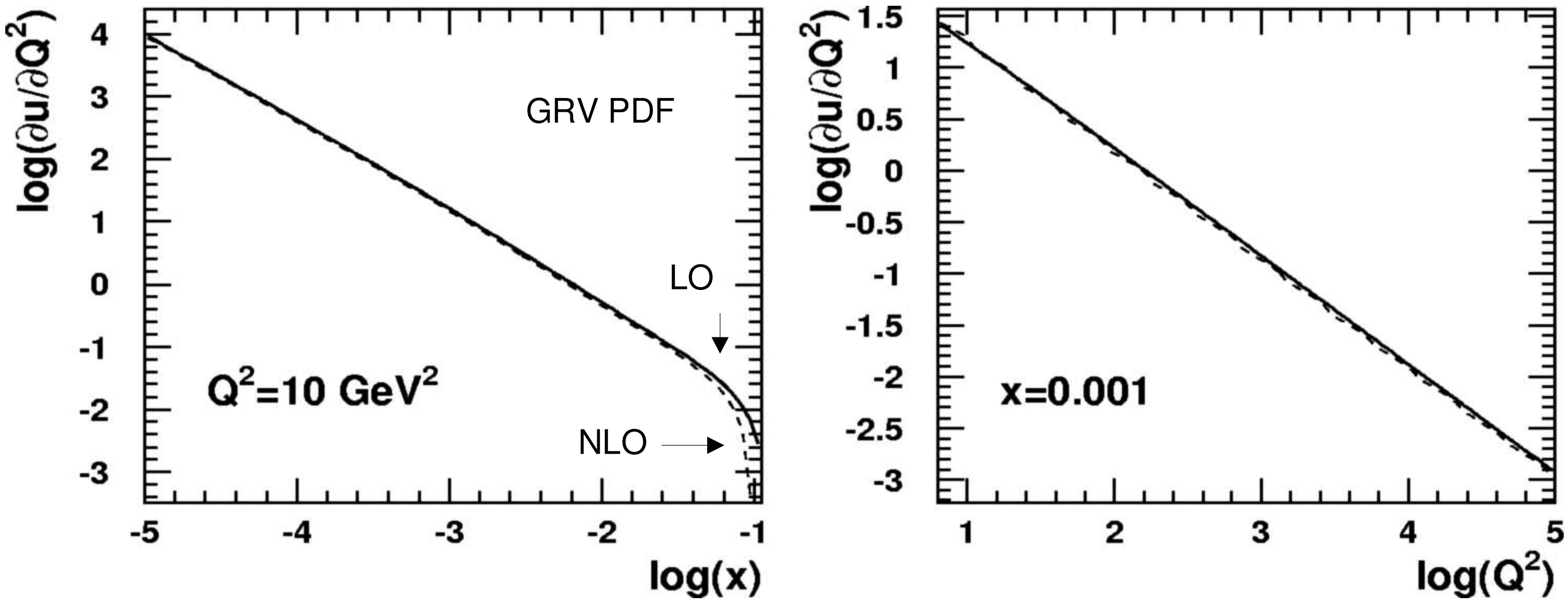
$$\log f(z) = D \log z + D_0$$

- **Multi–fractal:**

2 magnification factors *z*, *y*

$$\log f(z,y) = D_{zy} \log z \cdot \log y + D_z \log z + D_y \log y + D_0$$

Unintegrated Quark Densities



- **a hint for self–similar structure:**
  - **the unintegrated quark densities have linear behaviour in log–log plots versus x and Q^2**
- **two magnification factors: $1/x$ $1+Q^2/Q_0^2$**
- **integration needed to obtain conventional densities –> F_2**

- following the fractal dimension description of densities, a quark density may be written as

$$\log f_i(x,\ Q^2) = D_1 \log(1/x) \log(1+Q^2/Q_0^2) + D_2 \log(1/x) + D_3 \log(1+Q^2/Q_0^2) + D_0^i$$

where i denotes a quark flavour, $D_{\{1,2,3\}}$ are dimensional parameters and Q_0^2 is scaling of Q^2

- integrated densities: $q_i(x,Q^2) = \int_0^{Q^2} f_i(x,q^2)\, dq^2$

$$q_i(x\ ,\ Q^2) = \frac{e^{D_0^i}\, Q_0^2\, x^{-D_2}}{1+D_3-D_1 \log(x)} \left(x^{-D_1 \log(1+Q^2/Q_0^2)} \left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1} - 1 \right)$$

- F_2 is related to to the quark densities

$$F_2(x,Q^2) = x \sum_i e_i^2\, (q_i(x,Q^2) + \overline{q}_i(x,Q^2))$$

- thus assuming the flavour symmetry, F_2 may be expressed in the following analytical form

$$F_2(x, Q^2) = \frac{e^{D_0} Q_0^2 x^{-D_2+1}}{1+D_3-D_1 \log(x)} \left(x^{-D_1 \log(1+Q^2/Q_0^2)} (1+\frac{Q^2}{Q_0^2})^{D_3+1} - 1 \right)$$

- 5 parameters: D_0, Q_0^2, $D_{\{1,2,3\}}$

- Photoproduction limit, $F_2/Q^2 \to C(W^2)$ when $Q^2 \to 0$
  - $D_2=1$ gives exact convergence, fit to data preffers $D_2 \approx 1$ => possibility to fix D_2, 4 parameters
  - Bjorken *x* neglects masses at low W^2, low *x* -> additional mass term to *x*

$$x \to x + \frac{M^2}{W^2}$$

Fit to the F_2 Data

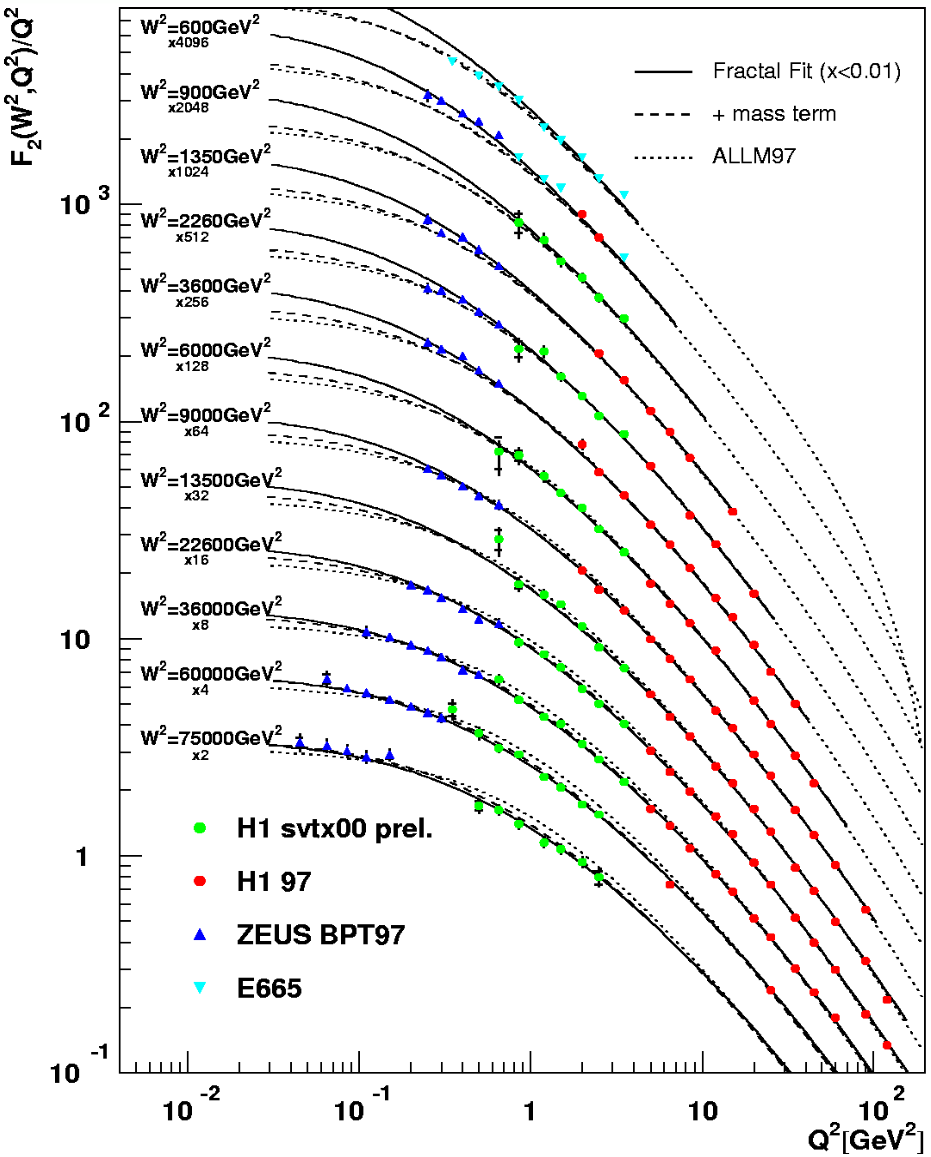
4 and 5 parameter fits:

$$\chi^2 / ndf = 0.82$$

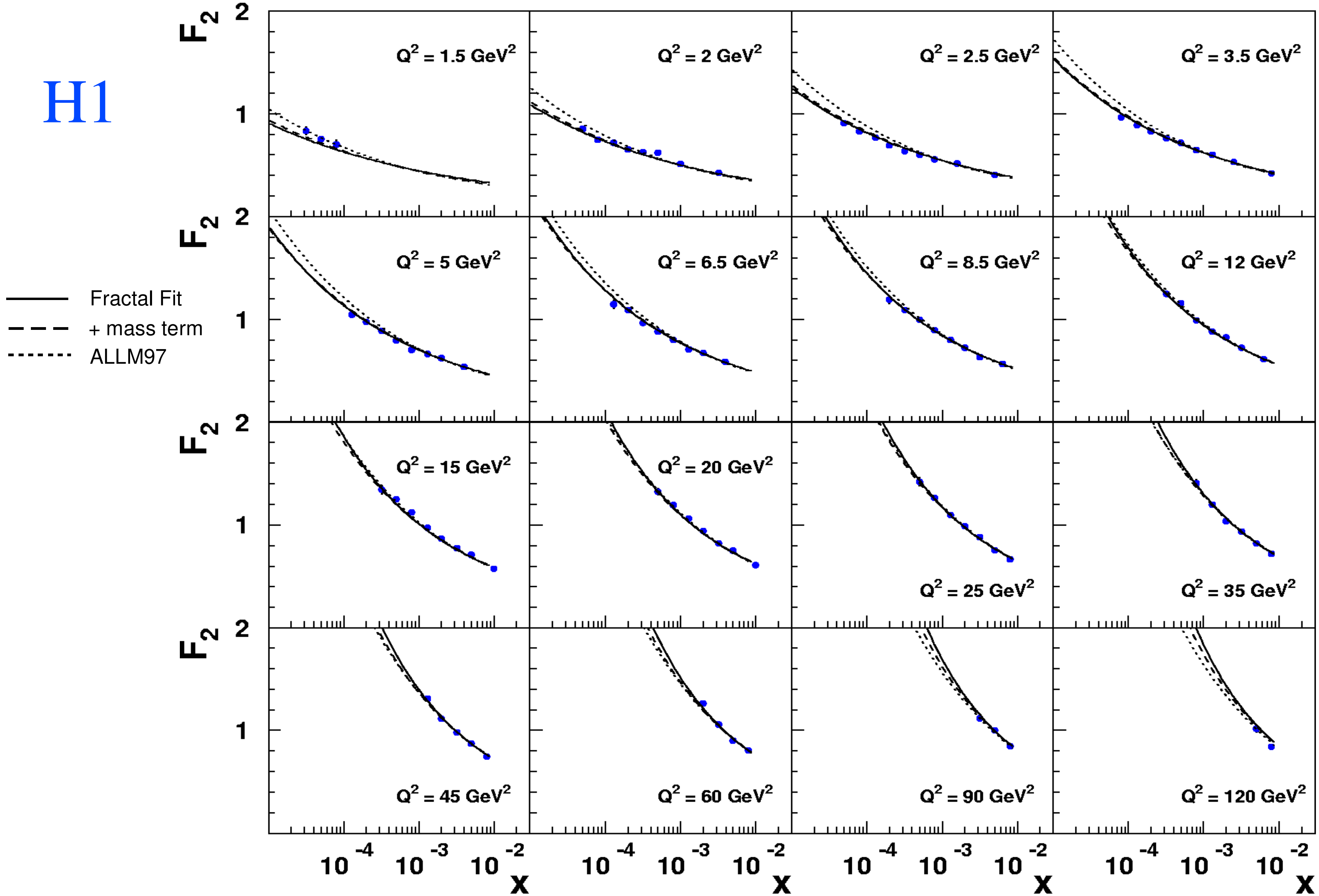
(172 points with x<0.01, total errors)

including the mass term the behaviour of the fit at low W^2 is similar to ALLM97 (i.e. below BPT points)

interesting to study the transition region $Q^2 \sim 1\ \mathrm{GeV}^2$ (difference of fits at higher W^2)

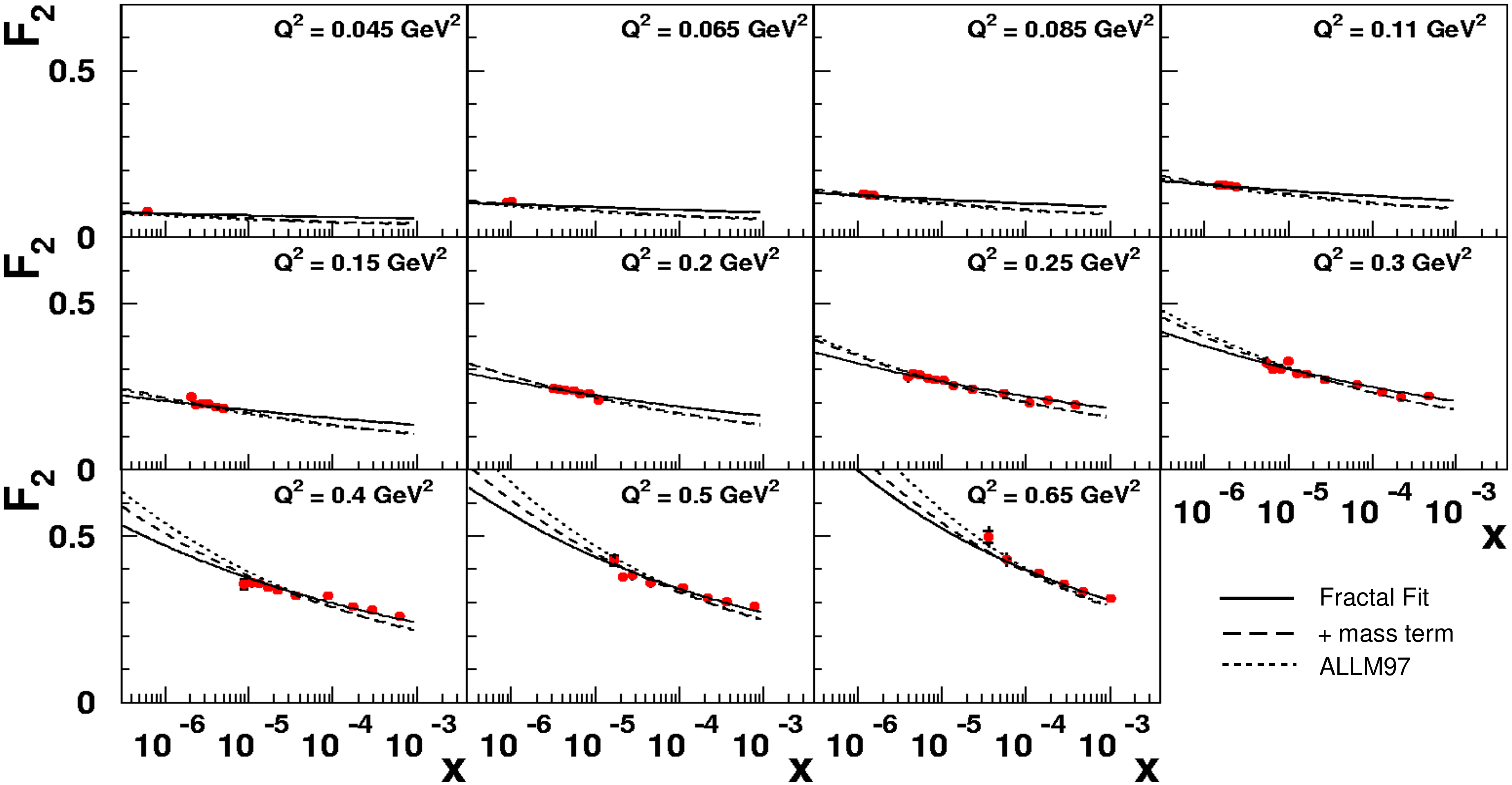


H1



F2
2
1
Q² = 1.5 GeV²
Q² = 2 GeV²
Q² = 2.5 GeV²
Q² = 3.5 GeV²
Q² = 5 GeV²
Q² = 6.5 GeV²
Q² = 8.5 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 20 GeV²
Q² = 25 GeV²
Q² = 35 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 90 GeV²
Q² = 120 GeV²
Fractal Fit
+ mass term
ALLM97
10⁻⁴ 10⁻³ 10⁻²
x

ZEUS



F2
0.5
0
Q^2 = 0.045 GeV^2
Q^2 = 0.065 GeV^2
Q^2 = 0.085 GeV^2
Q^2 = 0.11 GeV^2
Q^2 = 0.15 GeV^2
Q^2 = 0.2 GeV^2
Q^2 = 0.25 GeV^2
Q^2 = 0.3 GeV^2
Q^2 = 0.4 GeV^2
Q^2 = 0.5 GeV^2
Q^2 = 0.65 GeV^2
10^{-6} 10^{-5} 10^{-4} 10^{-3}
x
Fractal Fit
+ mass term
ALLM97

Summary

- **proton structure exhibits self−similar properties**
- **F_2 can be well described with fractal dimension approach, both in the non−perturbative and the deep inelastic domain**
- **extension in terms of other measures of the proton structure and kinematic regions is possible**
- hep−ph/0203260, subm. to EPJ C