On the Measurement of the Photon Structure of Protons in *ep* Collisions

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It was previously argued that QED Compton scattering (QEDC), $ep \longrightarrow e\gamma X$, at HERA can be employed for a measurement of the photon distribution function of the proton, which is defined in the framework of the equivalent photon approximation (Weizsäcker – Williams approach). It is shown that the equivalent photon approximation does not provide sufficient accuracy for the description of the inelastic QEDC scattering, and hence the proposed measurement of the cross section cannot be interpreted in terms of the photon density function.

1 Introduction

The equivalent photon approximation (EPA), also referred to as collinear approximation or Weizsäcker – Williams approximation [1, 2], is a commonly used method for the description of processes mediated by quasi-real photon exchange. In this approach the "slightly virtual" photon is treated as a real one, in particular, it is considered to be only transversely polarised. Furthermore the transverse component of its momentum is neglected, *i.e.* it is assumed to be emitted collinear to the incident particle, which leads to a reduction of the number of degrees of freedom in the final state. These assumptions allow a much easier and often more transparent calculation of the cross section. However the simplification of the formalism is achieved at the cost of worsening precision, and it is therefore necessary to prove the validity of the method by comparing it with exact computations. Various examples of EPA application to reactions in *ep* and *pp* collisions can be found in [3–12].

The quasi-real photon is considered in EPA as a constituent in the incident particle, similar to gluons and quarks in hadrons. The probability to detect it in the incident particle is given by the "structure" function γ, which thus determines the photon–parton content of the particle or its equivalent photon flux.

The possibility of measuring the photon content of the proton at HERA was first discussed in [3] and further considered in [5, 6, 11]. The QED Compton scattering was regarded as the reaction best suitable for such a measurement. The QEDC process represents one of three poles in the cross section of *ep* scattering with photon emission from the lepton side, described by the Feynman diagrams in fig. 1.

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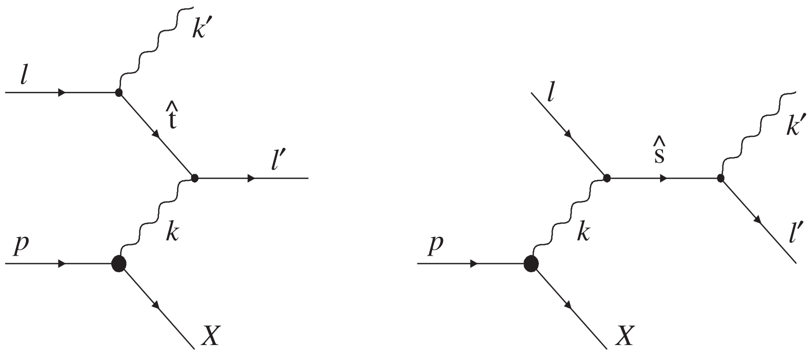

Fig. 1: Lowest order Feynman diagrams for the process $ep \to e\gamma X$ with the photon emision from the electron line. Letters denote particle four-momenta except for $\hat{t}$ and $\hat{s}$ which denote the corresponding four-momenta squared.

Each of the three poles corresponds to a distinct experimental configuration. The two poles, at which either $\hat{t}$ or $\hat{s}$ tends to zero while the exchanged photon virtuality stays finite, correspond to the so called Initial and Final State Radiation processes (ISR and FSR), where the photon is emitted collinear either to the incoming or the outgoing electron, respectively. The third peak, where the exchanged photon is almost real, while $\hat{t}$ and $\hat{s}$ are finite, describes QED Compton events with a characteristic final state topology: the outgoing electron and photon are detected under large polar angles almost back-to-back in azimuth. In this configuration their transverse momenta are nearly balanced, which leads to very low values of the exchanged photon virtuality.

The QEDC scattering offers a good possibility for a direct comparison of the collinear approximation with an exact computation of the cross section. A complete calculation has to take into account five non-trivial degrees of freedom, corresponding to energies and polar angles of the final electron and photon and to the azimuthal angle between them. Although such a calculation is more complex than the EPA approach, it is still feasible. It was performed by Courau and Kessler [13] using the helicity amplitude technique [14, 15].

The calculation embraces both elastic and inelastic scattering at the proton vertex. The photoabsorption cross section of the elastic scattering is described in terms of the proton electric and magnetic form-factors, G_E and G_M. The parameterisations for the inelastic scattering are provided separately for the region of resonances and the continuum domain.

The Monte Carlo generator COMPTON [16, 17] based on that calculation is employed in the present study. We use a new version of the generator [18] with improved cross section parameterisations in the resonance and continuum inelastic domains. To

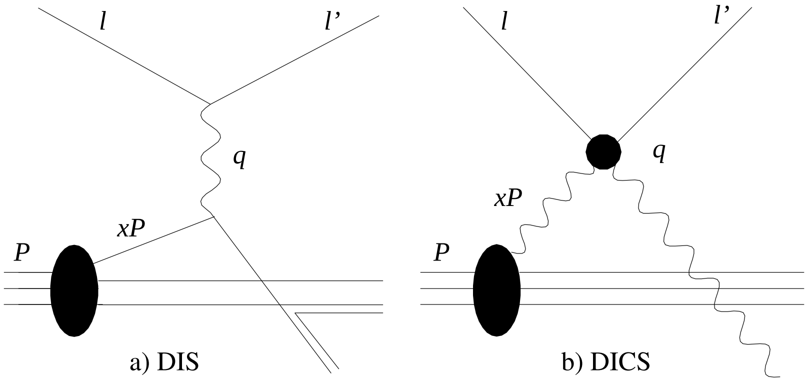

Fig. 2: Deep inelastic a) conventional and b) Compton scattering. Figure from [11].

be specific, the parameterisation by Brasse *et al.* [19] is employed to describe the resonance production, while the ALLM97 F_2 fit [20] is used for the continuum region.

To perform the comparison of the exact and approximative approaches we start with the proposal by De Rújula and Vogelsang [11] to measure the photon content of the proton and compare its Q^2 evolution with that of the gluon distribution function. In the following section the EPA calculation of the QEDC process and the consideration by De Rújula and Vogelsang are briefly described. Afterwards the results of our comparison of this calculation with the exact computation are presented.

2 QEDC Scattering in Collinear Approximation

The conventional deep inelastic scattering is usually treated as probing a quark in the proton by a virtual photon. Similarly, the QEDC scattering is considered in EPA as probing the photon in the proton by a virtual lepton, as schematically shown in fig. 2. The outgoing photon plays the role of the struck quark in DIS. Due to this analogy to the conventional DIS the QEDC process in this representation is called *Deep Inelastic Compton Scattering (DICS)*.

The variables naturally used for the kinematics definition in EPA are $Q^2 = l - l'$, $x = \frac{Q^2}{2P\cdot q}$ and $y = \frac{P\cdot q}{P\cdot l}$. Only two non-trivial degrees of freedom are necessary for the description of the differential DICS cross section. The cross section is given as a convolution of the probability $\gamma(x, Q^2)$ to find a photon with a momentum xP "inside" the proton by probing it at the scale Q^2 and the cross section of the Compton process $e\gamma \to e\gamma$ [21]:

$$\frac{d^2\sigma^{\rm DICS}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xs}\frac{1+(1-y)^2}{1-y}\gamma(x, Q^2)\ . \tag{1}$$

According to this expression, an experimental determination of the double differential QED Compton scattering cross section $d^2\sigma/dx\,dQ^2$ can be interpreted in the collinear approximation as a measurement of the photon–parton density function $\gamma(x, Q^2)$.

The γ function is calculated separately for elastic and inelastic QEDC events and the total function is the sum of both contributions: $\gamma = \gamma_{\rm el} + \gamma_{\rm in}$.

Like in the exact calculation mentioned above, the elastic contribution in EPA is written down in terms of the electric G_E and magnetic G_M form factors, and it turns out to depend only on x but not on Q^2 [4], *i.e.* there is no scaling violation for the elastic component.

For the inelastic scattering the γ function is given by the expression [6, 21]:

$$\gamma_{\rm in}(x, Q^2) = \frac{\alpha}{2\pi}\int_x^1 dz \int_{Q^2_{\rm min}}^{Q^2} \frac{dQ^2}{Q^2}\frac{z}{x}\left[\frac{1+(1-z)^2}{z^2}F_2\left(\frac{x}{z}, Q^2\right) - F_L\left(\frac{x}{z}, Q^2\right)\right] , \quad (2)$$

where the lower kinematic limit is given by

$$Q^2_{\rm min} = \frac{x^2 m_p^2}{1-x} . \quad (3)$$

Neglecting F_L and giving F_2 in terms of parton distribution functions one obtains:

$$\frac{\partial\gamma(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{d\zeta}{\zeta}\sum_{i=1}^{N_f} q_i(\zeta, Q^2) P_{\gamma q}\left(\frac{x}{\zeta}\right) , \quad (4)$$

where the quark-to-photon splitting function is:

$$P_{\gamma q}(z) = \frac{1+(1-z)^2}{z} . \quad (5)$$

The expression 4 can be called the "QED evolution equation" similar to the DGLAP gluon evolution equation in QCD [22–24]:

$$\frac{\partial g(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{d\zeta}{\zeta}\left[\sum_{i=1}^{N_f} q_i(\zeta, Q^2) P_{gq}\left(\frac{x}{\zeta}\right) + g(\zeta, Q^2) P_{gg}\left(\frac{x}{\zeta}\right)\right] \quad (6)$$

with the quark-to-gluon and gluon-to-photon splitting functions

$$P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z} \equiv \frac{4}{3}P_{\gamma q}(z) , \quad P_{gg}(z) = 6\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right) . \quad (7)$$

Equation (4) is in principle valid only in the region where F_2 can be represented in terms of parton distribution functions, *i.e.* at Q^2 beyond some typical lower pQCD scale Q_0^2. Like the QCD evolution equations it can be solved starting from some quark distribution at Q_0^2 and evolving it to higher Q^2 values. De Rújula and Volgelsang used

$Q_0^2 = 0.25\,\mathrm{GeV}^2$ as the start scale [25] and the parton density functions by Glück, Reya and Vogt [26].

The most essential difference between equations (4) and (6) is the self-coupling of gluons P_{gg} arising from their non-abelian nature. Consequently these two equations predict strikingly different Q^2 evolutions of γ and g functions.

One should note, that there is no elastic contribution to the gluon density function. But the (well-known) elastic contribution to γ is independent on Q^2, therefore there is no need to separate only the inelastic contribution in order to investigate its dependence on Q^2 and compare to the gluon content. The expected Q^2 behaviour of the γ function at some constant x values accessible at HERA is depicted in fig. 6 of [11] in comparison to the leading order gluon distribution function by Glück, Reya and Vogt [26].

3 Comparison of Computations

One should stress that the QEDC cross section measurement can be interpreted as a measurement of the function γ only if the collinear approximation is applicable in the studied kinematic region. However, a closer look at the QEDC event topology makes us perform a dedicated study of the accuracy of this approach. As an example, the distributions of the transverse momentum of the exchanged photon, p_{t,γ^*}, in events of elastic and inelastic scattering generated by the COMPTON program are presented in fig. 3. In the collinear approximation p_{t,γ^*} is assumed to be zero. One can see, that in elastic events it is really very small. On the contrary, in the inelastic events it is significantly larger. Consequently, the collinear approximation may lead to significant migrations of inelastic events between kinematic bins due to not sufficiently accurate calculation of event kinematics.

The effect may become especially strong due to the acceptance constraints, which demand that both outgoing particles, electron and photon, are measured under finite polar angles in the detector. In this study we impose the following cuts on the energies E_e, E_γ and polar angles θ_e, θ_γ of the final state electron and photon and on their azimuthal acoplanarity $A = \pi - \Delta\phi$:

$$\begin{aligned} &\min(E_e, E_\gamma) > 4\,\mathrm{GeV}\,, \quad E_e + E_\gamma > 20\,\mathrm{GeV}\,, \\ &0.06 < \theta_e, \theta_\gamma < \pi - 0.06\,, \quad A < 45^\circ\,. \end{aligned} \tag{8}$$

These limits are close to those used by De Rújula and Vogelsang and are chosen so to correspond approximately to the cuts, which whould be applied in a measurement at HERA.

In the following, a direct comparison between the exact calculation and the collinear approximation is described.

At first, a cross check of the γ function calculation done in [11] was performed using the above formalism. The elastic part of the function was calculated according to the

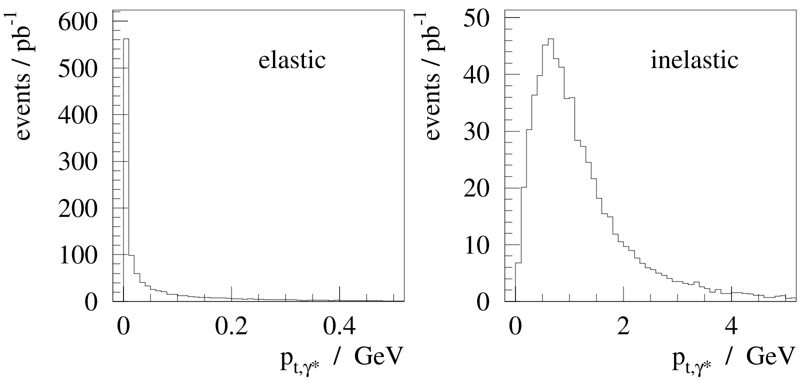

Fig. 3: Transverse momentum of the exchanged photon in elastic (left) and inelastic (right) QEDC events generated by the COMPTON program.

same expressions as employed by De Rújula and Vogelsang. In doing so exactly the same values were obtained. The inelastic component was computed according to eq. (2) by a numerical integration of the F_2 parameterisations, employed in the COMPTON generator. The integration over Q^2 was performed in eight x bins at the same x values as chosen by De Rújula and Vogelsang.

The result of the calculation is contrasted to the γ function by De Rújula and Vogelsang in fig. 4. Both calculations give very close results. The discrepancies between them are less than 4% in all bins. Slight deviations arise from the different F_2 parameterisations and from the choice of the integration start scale $Q^2_{\rm min}$ in eq. (2). In the calculation presented here the lower kinematic limit given by eq. (3) was set. This was possible, since the F_2 fits used in the present analysis are not based on pQCD and do not require artificial Q^2 limits. In order to test the influence of this parameter, the integration was performed once more, but with the start scale of $0.25\,{\rm GeV}^2$, as used by De Rújula and Vogelsang. The result is also displayed in fig. 4. As expected, the γ values have somewhat decreased, but the general Q^2 behaviour remained unchanged.

In the second step the leading order cross section values obtained by the exact approach were compared with those calculated in the collinear approximation. To perform the comparison the QEDC cross section was computed from the sample of COMPTON events generated without radiative corrections. The values were determined in x–Q^2 bins by dividing the total number of events fulfilling the selection criteria specified above by the effective luminosity of the MC sample. The cross section in collinear approximation was computed analytically by W. Vogelsang [25] with the same set of cuts in the same bins. The uncertainty of the analytical calculation equals approximately to 2% in all bins. As can be seen in fig. 5, there are significant discrepancies in several bins between the generated values and those calculated in EPA.

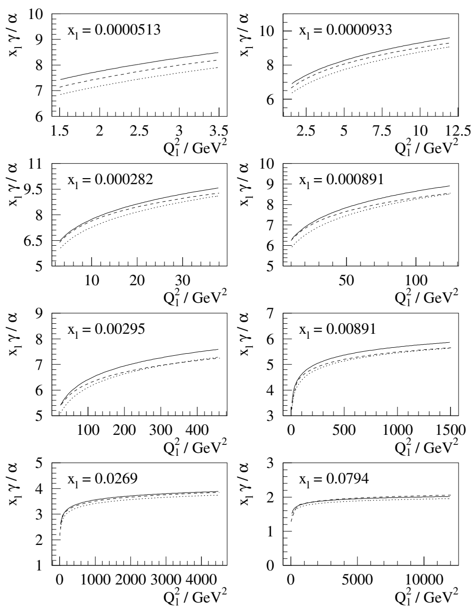

Fig. 4: γ function at constant x values. The solid line depicts the function calculated in the present analysis. The dotted line shows the same calculation, but with the lower integration limit of the inelastic contribution set to $Q_0^2 = 0.25\,\mathrm{GeV}^2$. The dashed line displays the γ function computed in [11].

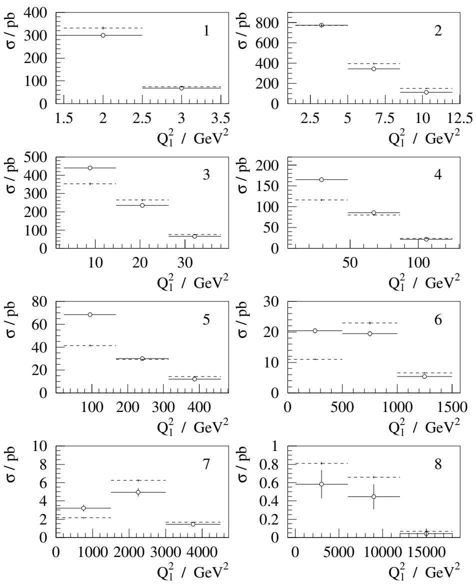

Fig. 5: Double differential cross section of QED Compton scattering. Open circles depict values given by the COMPTON generator. The corresponding error bars show statistical errors. The dashed lines denote the values computed by W. Vogelsang.

Furthermore the elastic scattering cross section values generated by COMPTON and computed in the Weizsäcker – Williams approximation were compared and a very good agreement was obtained. Here one should bear in mind that the same electric and magnetic form factors of the proton have been employed both for the exact treatment in the COMPTON program and for the calculation in the equivalent photon approximation. Hence, this approximation provides a very good description of the elastic scattering.

The source of the discrepancies is, thus, as expected, in the treatment of the inelastic QEDC process. Since the γ functions agree with each other within 4%, but the cross sections computed in the collinear approximation exhibit an up to 25% deviation in several bins from those calculated by the exact method in COMPTON, we conclude that apparently the equivalent photon approximation does not supply a sufficient degree of accuracy in the inelastic region. In other words, the factorisation of the cross section given by eq. (1) in terms of only two kinematic variables is a too rough approximation of the real QEDC scattering process in the inelastic domain.

It is not ruled out that some carefully chosen cuts on the transverse momenta, polar angles and/or other event quantities may limit the phase space so that the equivalent photon approximation becomes applicable also in the inelastic QEDC scattering. However, it should be studied whether such limited phase space would still have experimental relevance for current detectors.

Instead, a more ambitious goal would be a direct measurement of F_2 using inelastic QEDC events at very low photon virtualities, *i.e.* in the transition regime from DIS to photoproduction.

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