QCD Fits & Extraction of PDFs

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on behalf of the H1 Collaboration

What do we learn from

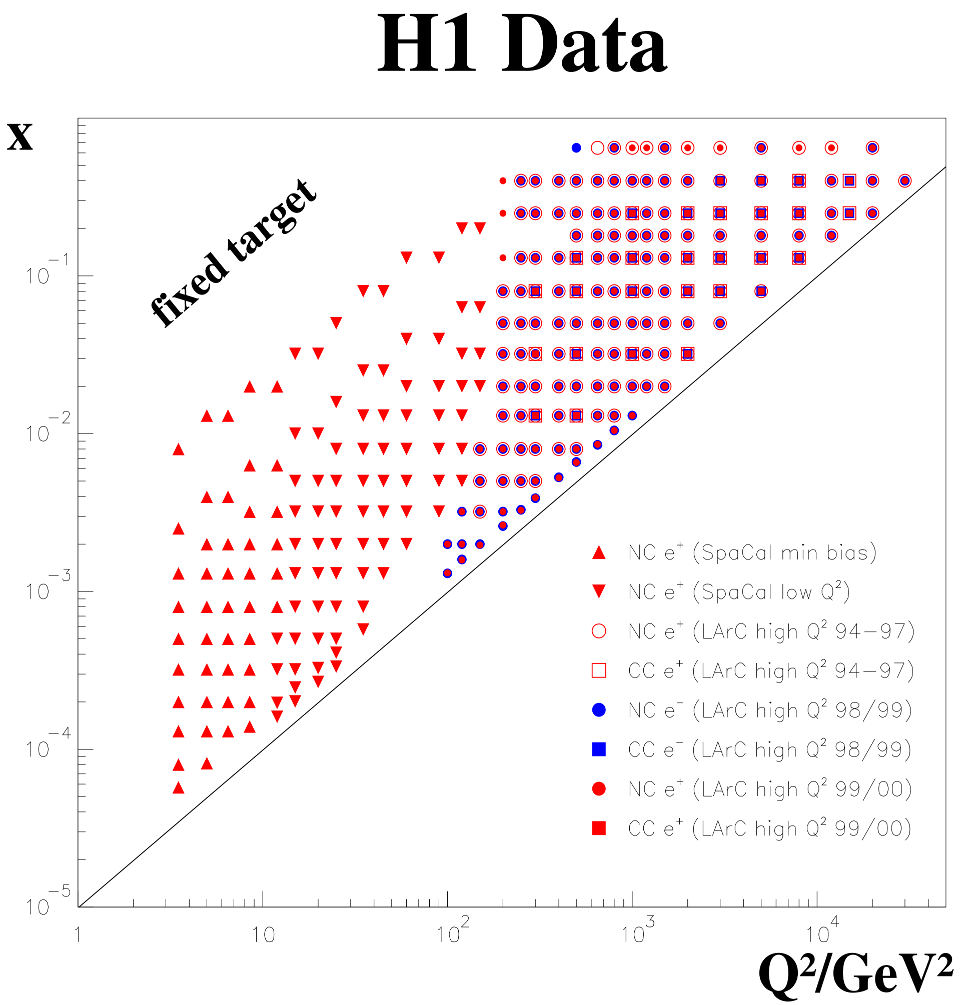
H1 DATA

of HERA Run I?

- H1 Inclusive Data
- The New H1 PDF Fit
- Results of the Fit

(for dedicated α_s & *gluon* fit
see previous DIS workshops)

H1 Inclusive Data Sets



NC Low Q^2 e^+p(96-97)	hep-ex/0012053	EPJ C21 (2001) 33
NC&CC e^+p (94-97)	hep-ex/9908059	EPJ C13 (2000) 609
NC&CC e^-p (98-99)	hep-ex/0012052	EPJ C19 (2001) 269
NC&CC e^+p (99-00)	hep-ex/0304003	submitted to EPJ

Data used in published H1 fits:

– Low Q^2 NC e^+p (96-97)
  $1.5 < Q^2 < 150$ GeV^2
– High Q^2 e^+p (94-97)
  NC: $150 < Q^2 < 30000$ GeV^2
  CC: $300 < Q^2 < 15000$ GeV^2

New H1 Datasets:

- NC at $150 < Q^2 < 30000$ GeV^2
  e^-p 98-99 published
  e^+p 99-00 submitted
  high y, $y = 0.75$, $100 < Q^2 < 800$ GeV^2
  e^-p 98-99 & e^+p 99-00 submitted
- CC at $300 < Q^2 < 15000$ GeV^2
  e^-p CC 98-99 published
  e^+p CC 99-00 submitted

Experimental Uncertainties

High quality of H1's NC & CC inclusive data requires careful treatment of experimental uncertainties.
(see talk of Ana Dubak)

Statistical & Systematic Errors

$Q^2 \lesssim 100$ GeV^2:
stat. $\simeq 1\%$, syst. $\lesssim 3\%$
only for $Q^2 \gtrsim 1000$ GeV^2 :
stat. $>$ syst. uncertainty

Correlated Syst. Uncertainties:

$\delta^{\mathcal{L}}$:	luminosity uncertainty
δ^{E}:	electron energy calibration
δ^{θ}:	polar angle of electron
δ^{h} :	hadronic energy calibration
δ^{N}:	noise suppression algorithm
δ^{B}:	background estimate by Monte Carlo
δ^{V}:	cut against γp background (CC)
δ^{S}:	background subtraction using wrong charged lepton candidates (NC, high y)

Correlations between H1 Data Sets of different years taken into account

Year to Year Correlations of Systematic Errors

This table is given to complete the file on the conference webpages not to be shown during the presentation

data set	process	$\delta^{\mathcal{L}}$	δ^{E}	$\delta\theta$	δ^{h}	δ^{N}	δ^{B}	δ^{V}	δ^{S}
H1 minimum bias 97	e^+p NC	$\mathcal{L}1$	E1	$\theta1$	h1	N1	B1	–	–
H1 low Q^2 96-97	e^+p NC	$\mathcal{L}2$	E1	$\theta1$	h1	N1	B1	–	–
H1 high Q^2 94-97	e^+p NC	$\mathcal{L}3$	E2	$\theta2$	h2	N1	B2	–	–
H1 high Q^2 94-97	e^+p CC	$\mathcal{L}3$	–	–	h2	N1	B2	V1	–
H1 high Q^2 98/99	e^-p NC	$\mathcal{L}4$	E2	$\theta3$	h2	N1	B2	–	S1
H1 high Q^2 98/99	e^-p CC	$\mathcal{L}4$	–	–	h2	N1	B2	V2	–
H1 high Q^2 99/00	e^+p NC	$\mathcal{L}5$	E2	$\theta3$	h2	N1	B2	–	S1
H1 high Q^2 99/00	e^+p CC	$\mathcal{L}5$	–	–	h2	N1	B2	V2	–

NC Cross Section and Structure Functions

Inclusive NC Cross Section (γ^*, Z^0 exchange)

$$\frac{d^2\sigma^{e^\pm p}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\,(\underbrace{Y_+\tilde{F}_2(x,Q^2)\mp Y_- x\tilde{F}_3(x,Q^2) - y^2 F_L(x,Q^2)}_{\text{structure function term } \Phi^\pm_{NC}})$$

$$Y_\pm(y) = 1 \pm (1-y)^2$$

Generalized Structure Functions in Quark Parton Model:

$$\tilde{F}_2 = \sum_i A_i(Q^2)\,[xq_i + x\bar{q}_i] \Rightarrow F_2^{em} = \tfrac{4}{9}x(u+\bar{u}+c+\bar{c}) + \tfrac{1}{9}x(d+\bar{d}+s+\bar{s})$$

$$x\tilde{F}_3 = \sum_i B_i(Q^2)\,[xq_i - x\bar{q}_i] \Rightarrow x\tilde{F}_3 = B_U(u+c-\bar{u}-\bar{c}) + B_D(d+s-\bar{d}-\bar{s})$$

Electroweak Coefficient Functions $A_i(Q^2)$, $B_i(Q^2)$ (QED: $A_i = e_i^2$)

Sensitivity to (anti) quark types, not individual quark flavors

$\Rightarrow$ **Express Structure Functions in Terms of "Quark-Type-Functions":**

$$F_2^{em} = \tfrac{4}{9}x(U+\bar{U}) + \tfrac{1}{9}x(D+\bar{D})$$

$$x\tilde{F}_3 = B_U(U-\bar{U}) + B_D(D-\bar{D})$$

Charged Current and Deuteron Data

Charged Current Cross Section:

$$\frac{d^2\sigma^{\pm}_{CC}}{dx\,dQ^2} = \frac{G_F^2}{2\pi x}\left(\frac{{M_W}^2}{{M_W}^2+Q^2}\right)^2 \underbrace{(Y_+ W_2(x,Q^2) \mp Y_- x W_3(x,Q^2) - y^2 W_L(x,Q^2))}_{\Phi^{e^\pm p}_{CC}(x,Q^2)}$$

G_F Fermi coupling constant, M_W mass of $W^\pm$ Boson

Charged Current Structure Function Terms:

$$\Phi^{e^-p}_{CC}|_{LO} = xU + (1-y)^2 x\bar{D}$$
$$\Phi^{e^+p}_{CC}|_{LO} = x\bar{U} + (1-y)^2 xD$$

U, D, $\bar{U}$, $\bar{D}$ fully define the $e^\pm p$ NC, CC cross sections!

Deuteron Data Nucleon Structure Function ($\frac{1}{2}F_2^d = F_2^N$):

$$F_2^N = \frac{5}{18}x(U + \bar{U} + D + \bar{D}) + \frac{1}{6}(c + \bar{c} - s - \bar{s})$$

Assumptions & Constraints

Parametrize: xg, $xU = x(u+c)$, $xD = x(d+s)$, $x\bar{U}$, $x\bar{D}$

$$xP = A_P x^{B_P}(1-x)^{C_P}(1 + D_P x + E_P x^2 + F_P x^3 + G_P x^4)$$

Flavor Decomposition of "Quark Type Functions" at Starting Scale:

$\bar{c} = f_c \cdot \bar{U}$ (const. fraction HERA F_2^c);

$\bar{s} = f_s \cdot \bar{D}$ (const. fraction NuTeV)

assume symmetric quark-antiquark-sea ($c = \bar{c}$, $s = \bar{s}$)

Constraints:

Momentum sum rule: $\int_0^1 x(g + U + D + \bar{U} + \bar{D})\mathrm{d}x = 1$

u counting rule: $\int_0^1 (U - \bar{U})\mathrm{d}x = 2$

d counting rule: $\int_0^1 (D - \bar{D})\mathrm{d}x = 1$

Low x behavior:

$$\underbrace{xu_v = x(U - \bar{U}) \xrightarrow{x\to 0} 0}_{A_U = A_{\bar{U}},\ B_U = B_{\bar{U}}}; \quad \underbrace{xd_v = x(D - \bar{D}) \xrightarrow{x\to 0} 0}_{A_D = A_{\bar{D}},\ B_D = B_{\bar{D}}}; \quad \underbrace{x(\bar{d} - \bar{u}) \xrightarrow{x\to 0} 0}_{B_{\bar{U}} = B_{\bar{D}},\, A_{\bar{U}} = A_{\bar{D}}\frac{1-f_s}{1-f_c}}$$

New H1 FIT for Extraction of PDFs

Ingredients of the Fit

- Fit includes latest H1 high Q^2 data
- All inclusive H1 data
  NC & CC, e^+p & e^-p
  (BCDMS $p+d$)
- PDFs at starting scale
  $x\,g$, xU,$x\,D$,$x\bar{U}$,$x\bar{D}$
- Medium and high Q^2
  $3.5\ \mathrm{GeV}^2 < Q^2 < 30\,000\ \mathrm{GeV}^2$
- Massless Parton Scheme
- α_s fixed (dedicated α_s&g fit)
- Full treatment of experimental errors

Fitting Strategy

- additional parameter is only considered only if its introduction significantly improves the χ^2.
- add in only one new parameter at a time.

Error Bands

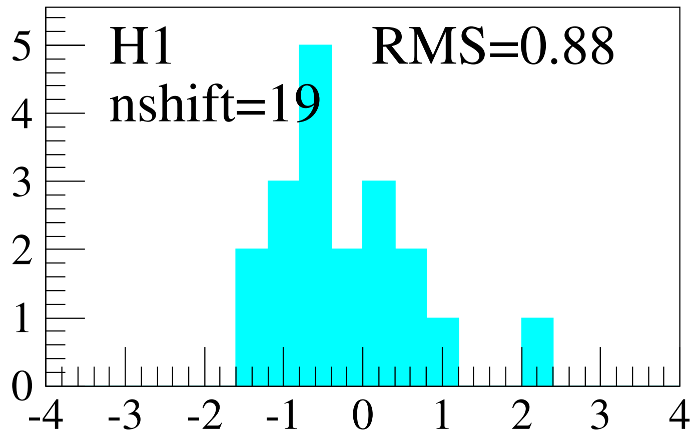
- **Experimental Error:**
  vary fitted parameter under constraint $\boldsymbol{\Delta\chi^2 \leq 1}$
- **Model Uncertainties:**
  Variation of
  Q^2_{min}, Q^2_0, α_s, f_s, f_c,m_c, m_b

Definition of χ^2

$$\chi^2 = \sum_k^{n_{exp}} \sum_i^{N_k} \frac{\left(\sigma_{k,i}^{exp} - \sigma_{k,i}^{th} \cdot (1 - n_k \delta_k^{norm} - \sum_j^{n_s(k)} s_{k,j}\, \delta_{k,i,j}^{syst})\right)^2}{\delta_{k,i}^{sta\,2} + \delta_{k,i}^{unc\,2}}$$

$$+ \sum_k^{n_{exp}} n_k^2 + \sum_k^{n_{exp}} \sum_j^{n_{s(k)}} s_{j,k}^2$$

- We explicitly allow the data points to be shifted by the fit



$\sigma_{k,i}^{exp}$:	measurement i of dataset k	$\delta_{k,i}^{unc}$:	uncorrelated systematic error
$\sigma_{k,i}^{th}$:	prediction for data point i of dataset k	$\delta_{k,i,j}^{sys}$:	correlated systematic error of source j
n_{exp}:	number of datasets	δ_k^{norm}:	uncertainty of dataset normalization
N_k:	number of data points in dataset k	n_k:	shift of normalization
$\delta_{k,i}^{sta}$:	statistical error	$s_{k,j}$:	shift of systematic of source j

"Pascaud and Zomer" method, extended χ^2 (preprint LAL-95-05):

Results: Functional Forms

Functional Forms of PDFs

in Fit to H1 data

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} [1 + D_g x]$$
$$xU(x) = A_U x^{B_U} (1-x)^{C_U} [1 + D_U x + F_U x^3]$$
$$xD(x) = A_D x^{B_D} (1-x)^{C_D} [1 + D_D x]$$
$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}}$$
$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$$

10 free parameter

$$\boldsymbol{\chi^2/ndf = 540/(621-10) = 0.88}$$

as expected:

- most sensitive to U
- H1 + BCDMS p & d require more flexibility at high x

in Fit to H1 + BCDMS p & d data

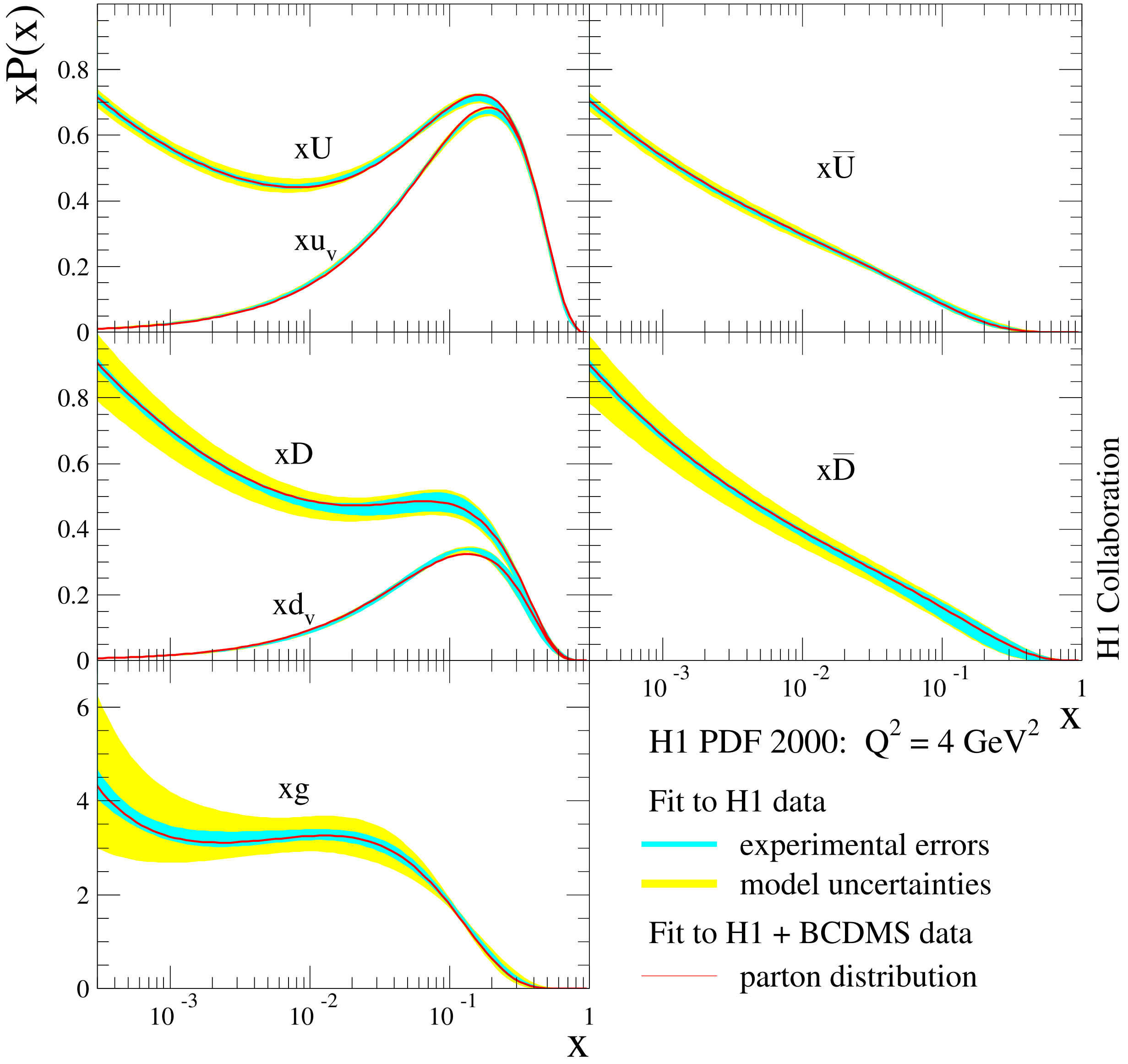
$$xU(x) = A_U x^{B_U} (1-x)^{C_U} [1 + D_U x + F_U x^3 + G x^4]$$
$$xD(x) = A_D x^{B_D} (1-x)^{C_D} [1 + D_D x + F x^3]$$

g, $\bar{U}$, $\bar{D}$ functional same as in H1 fit

12 free parameter

$$\boldsymbol{\chi^2/ndf = 883/(1014-12) = 0.88}$$

Fit Results: Parton Distributions



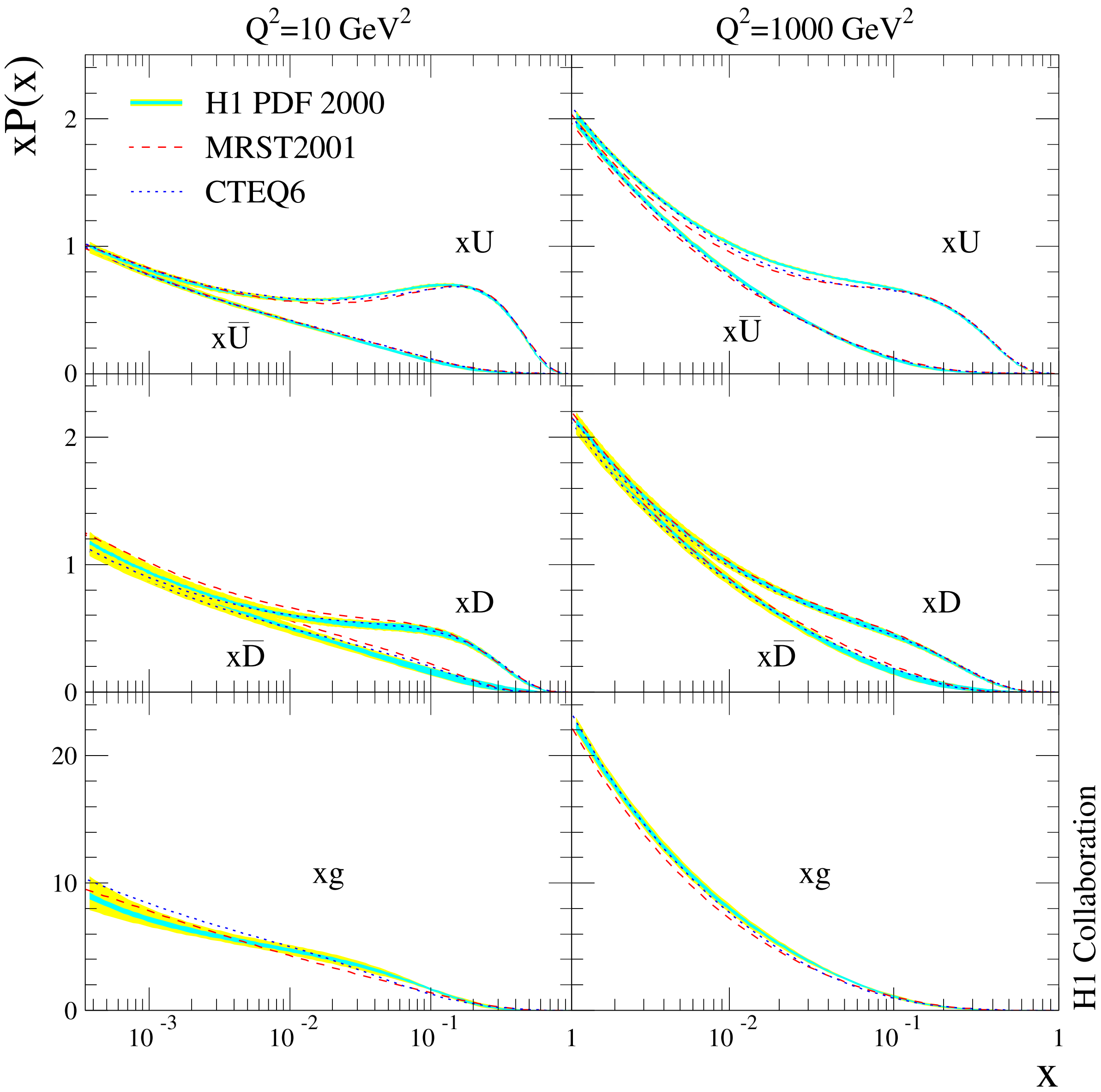
- PDFs from H1 and H1 + BCDMS p & d agree very well

- exp. uncertainties of H1 PDFs

x	0.01	0.4	0.65
xU	1%	3%	7%
xD	2%	10%	30%

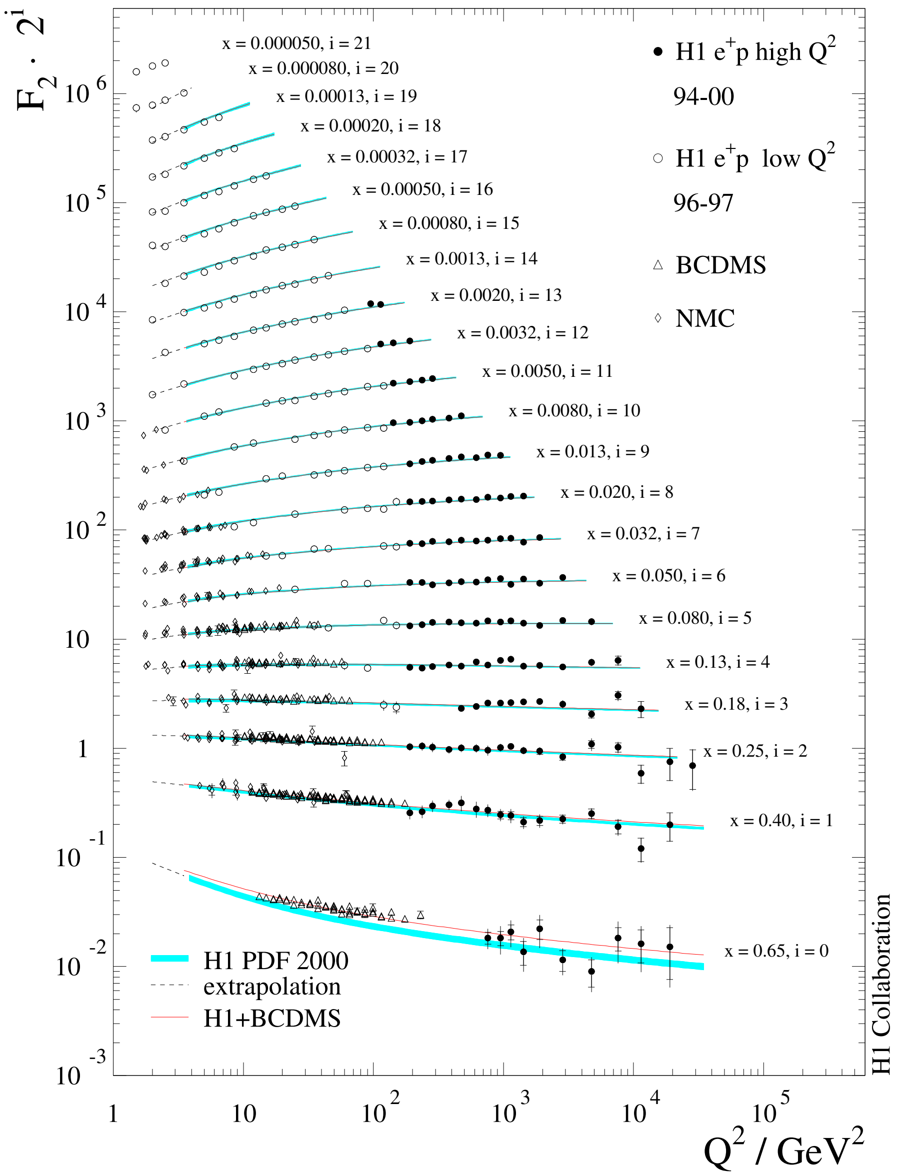
- note errors at low x depend significantly on fit assumptions

Comparison to Global Fits



H1 PDF 2000 fit is
in remarkable agreement
with global Fits
MRST and CTEQ

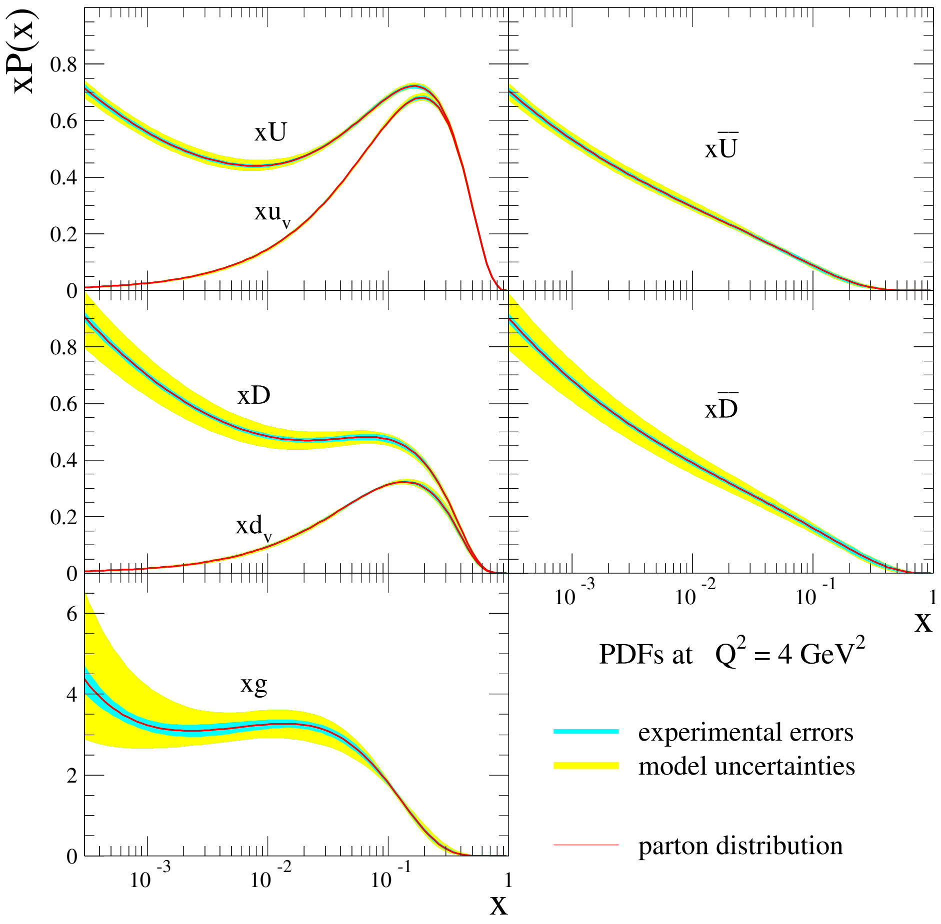
Structure Function F_2



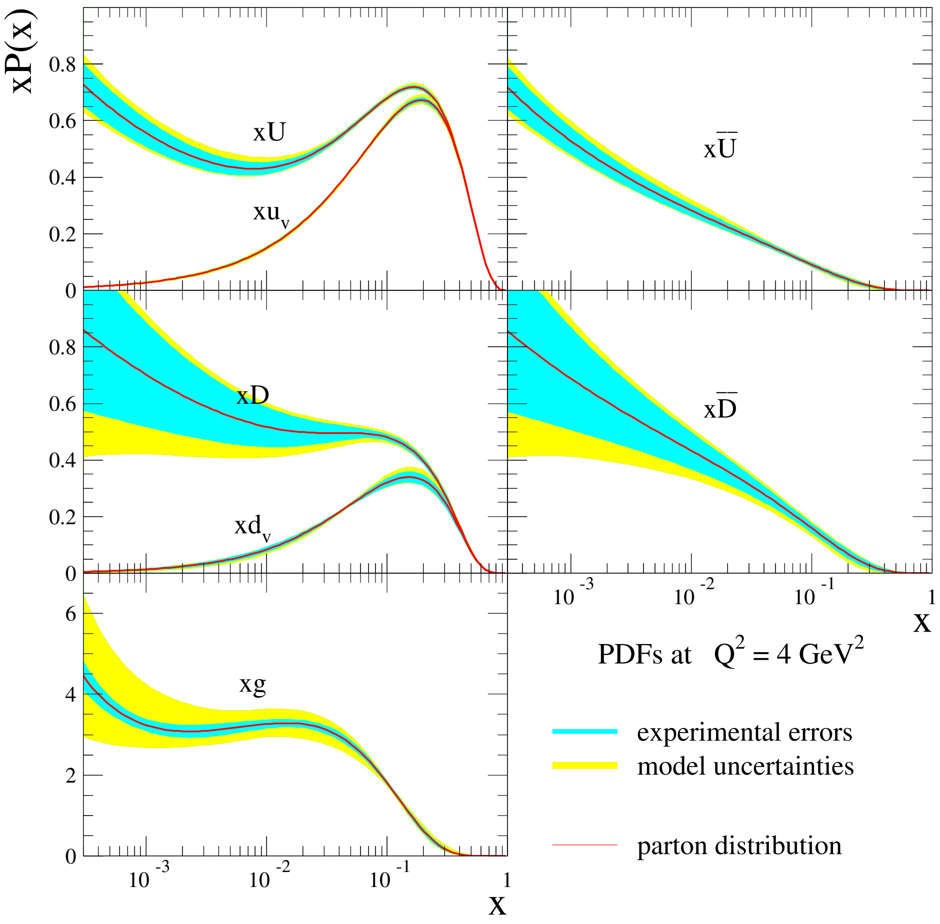
- H1 data cover a range of four orders of magnitude in x and Q^2
- H1 PDF 2000 fit provides good description of the data over whole region, except BCDMS data at $x = 0.65$
- Fit describes well the strong scaling violations

Impact of Low x Constraint

H1 + BCDMS p & d



H1 + BCDMS p & d (free $A_{\bar{U}}$, $B_{\bar{U}}$)



enforce

$x(\bar{d} - \bar{u}) \xrightarrow{x \to 0} 0$

constraints: $A_{\bar{U}} = A_{\bar{D}} \frac{1-f_s}{1-f_c}$,

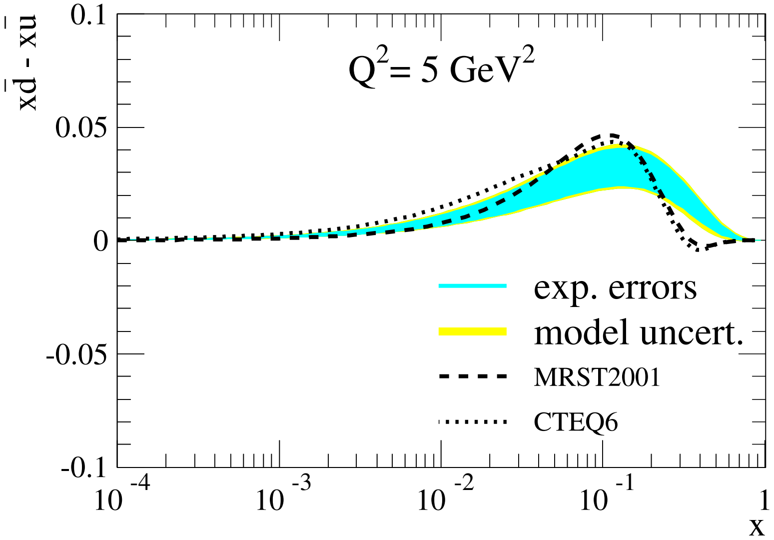
$B_{\bar{U}} = B_{\bar{D}}$

$A_{\bar{U}}$ and $B_{\bar{U}}$

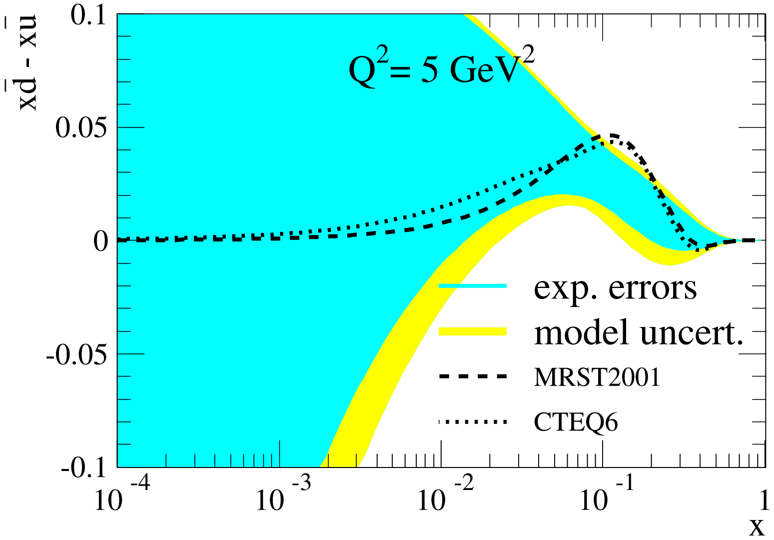
free parameters

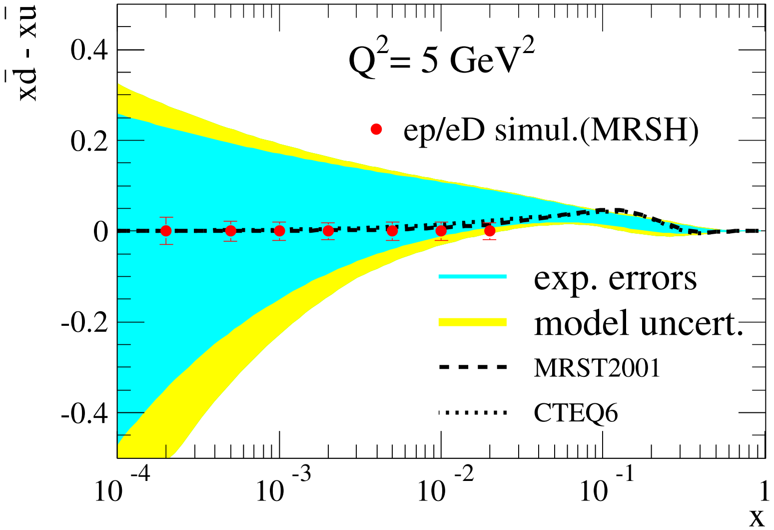
Impact of Low x Constraint: $\bar{d} - \bar{u}$

H1 + BCDMS p & d



H1 + BCDMS p & d (free $A_{\bar{U}}$, $B_{\bar{U}}$)





- shape of $\bar{d} - \bar{u}$ in global fits reproduced by fit to H1 + BCDMS p & d when $x(\bar{d} - \bar{u}) \xrightarrow{x \to 0} 0$ is imposed
- uncertainty is much wider when this constraint is not applied
- test of symmetry of light sea quarks at low x requires NEW data (p&d)

Summary

- Highlights of the New H1 PDF Fit
  - determination of the PDFs and their uncertainties from one single experiment: H1 NC & CC
  - novel decomposition of quark species into up and down type quark distributions

    $x\,g,\ xU, x\,D, x\bar{U}, x\bar{D}$
  - parameter space narrowed using theoretical constraints adopted to the new ansatz

comments

- stringent constraints at high x requires more statistics
- very rough strange and charm assumptions
- test of light sea symmetry at low x requires NEW data (p&d)