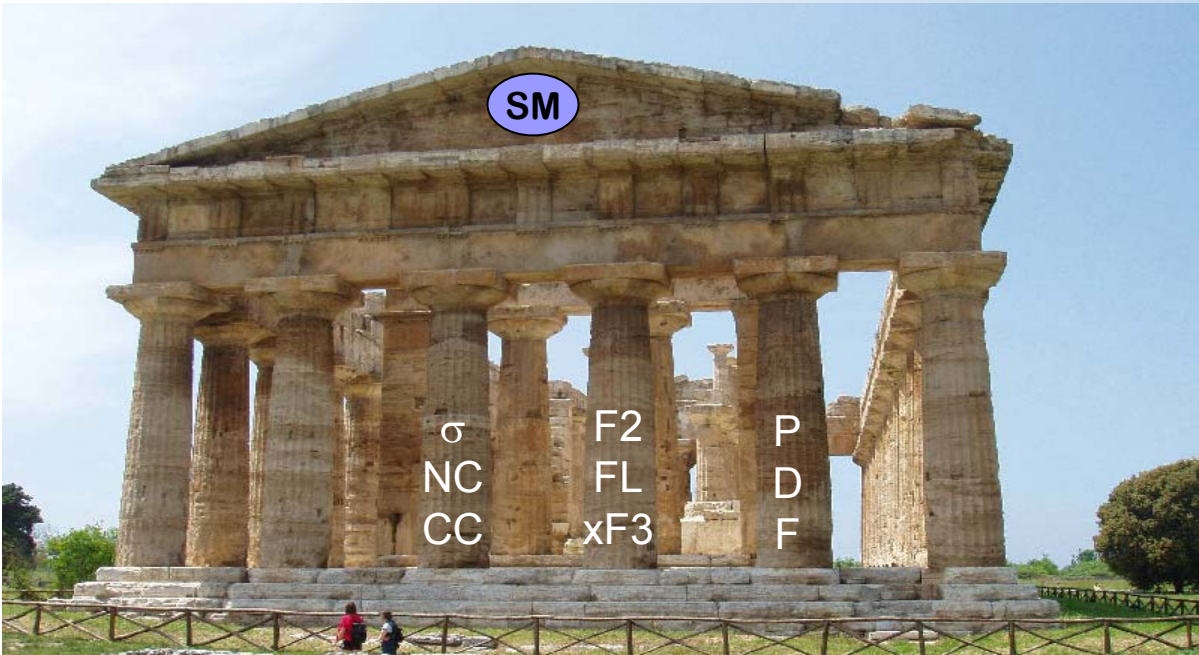


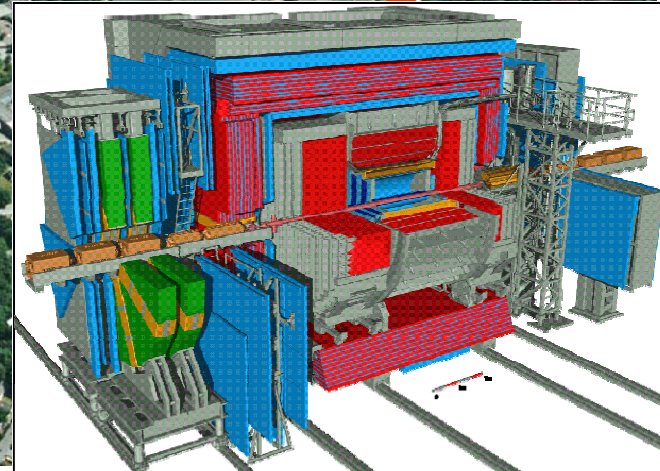
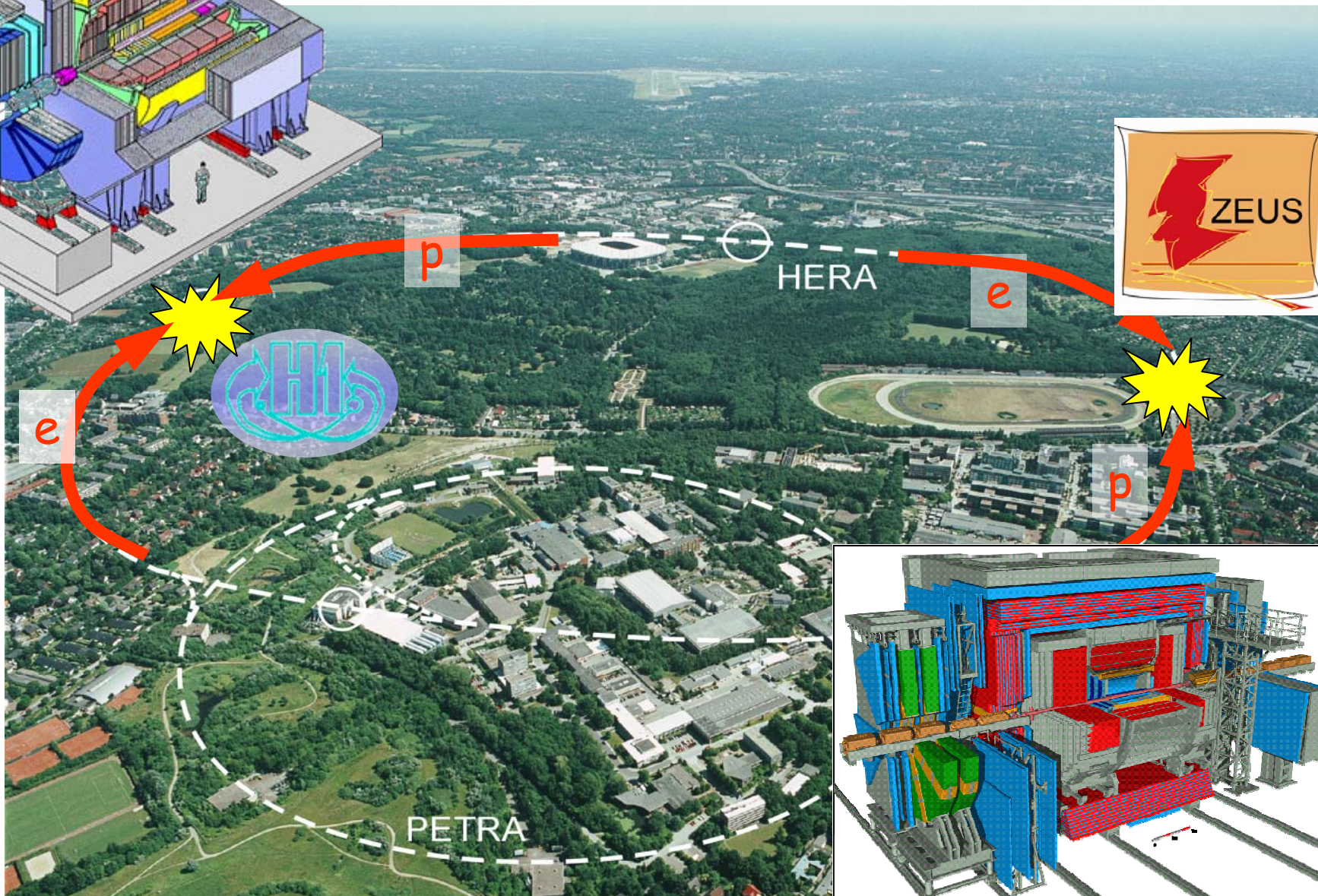
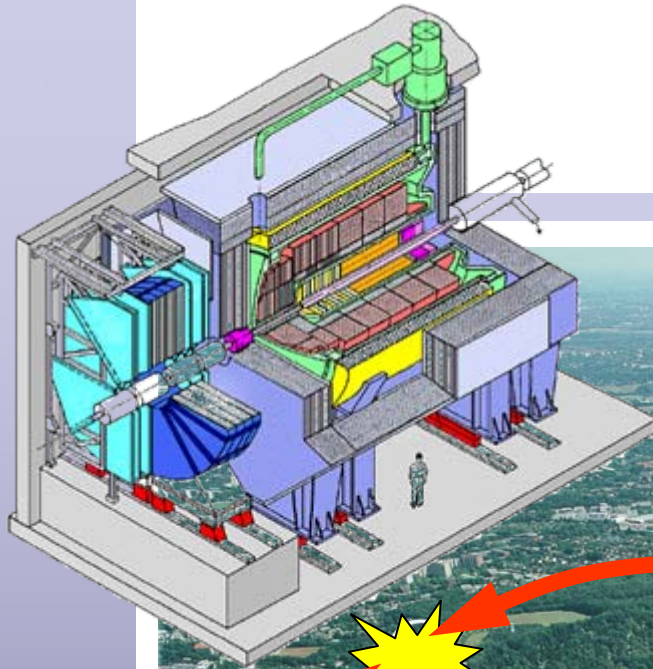


F_2 and PDFs at HERA

Building up the HERA Legacy:



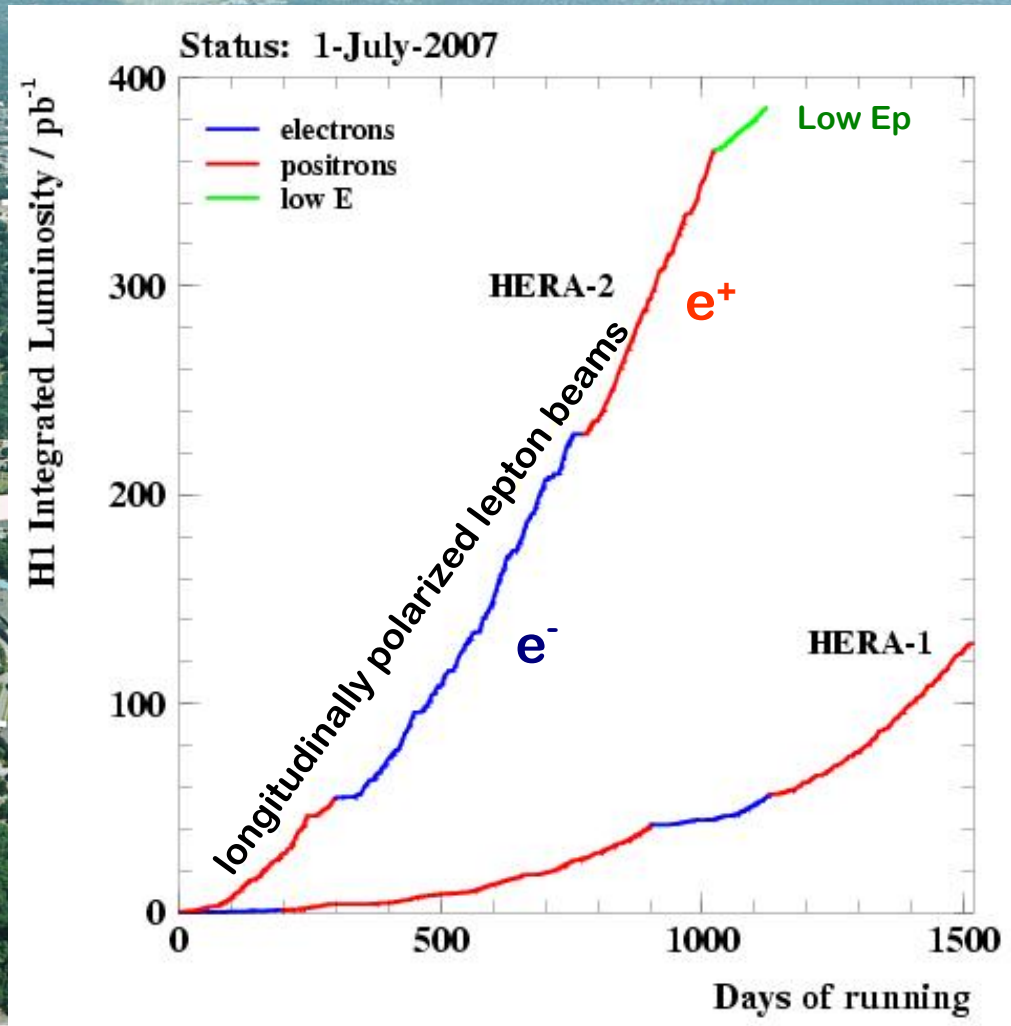
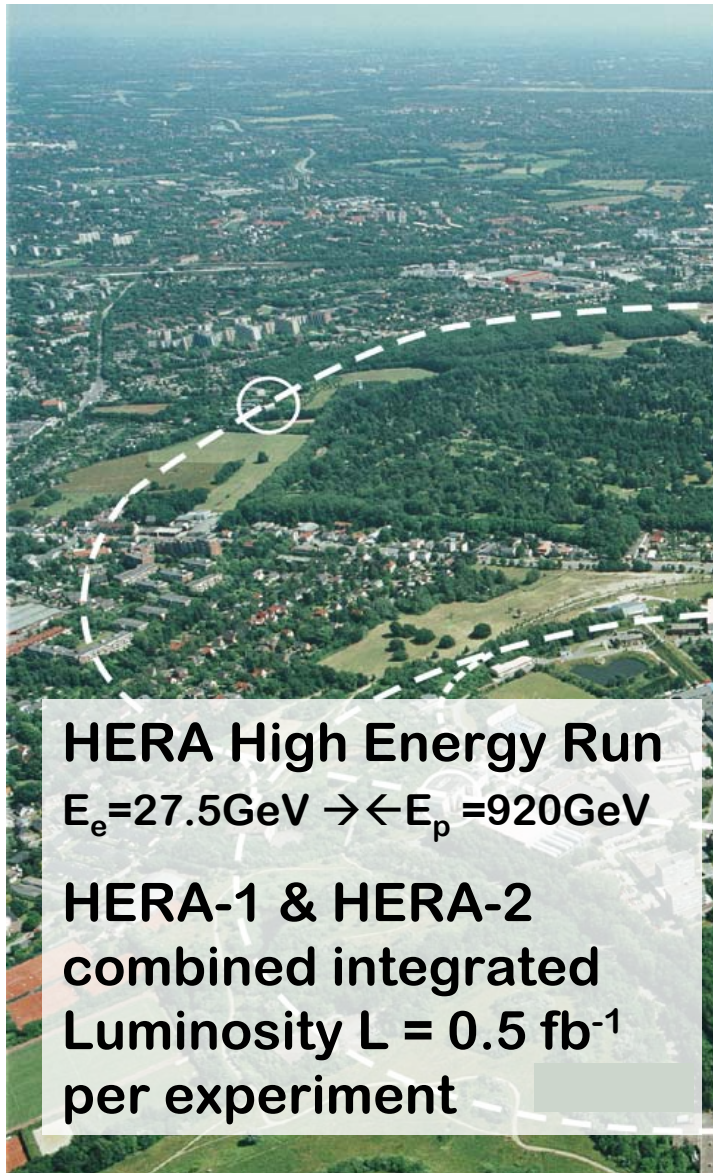
HERA Accelerator



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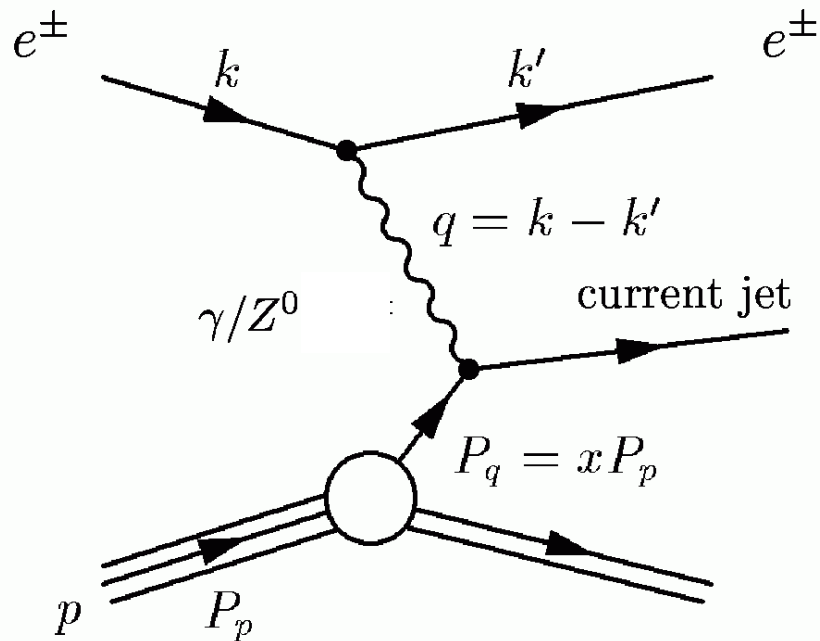
HERA Accelerator Performance



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Deep Inelastic Scattering



Kinematic Variables

- 4-momentum transfer resolving power

$$Q^2 = -q^2 = -(k - k')^2$$

- Björken scaling variable momentum fraction of struck parton

$$x = \frac{Q^2}{2p \cdot q}$$

- Inelasticity:

$$y = \frac{p \cdot q}{p \cdot k}$$

Center of mass energy \sqrt{s} : $s = (k + p)^2$ relation for fixed s: $Q^2 = sxy$

- Neutral current DIS cross section expressed by structure functions:

$$\frac{d^2 \sigma^{e^\pm p \rightarrow e^\pm X}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \underbrace{\left(1 + (1-y)^2\right)}_{Y_\pm = 1 \pm (1-y)^2} \cdot \left(\boxed{F_2(x, Q^2)} - \frac{y^2}{Y_+} \boxed{F_L(x, Q^2)} \mp \frac{Y_-}{Y_+} \boxed{x F_3(x, Q^2)} \right)$$

valence & sea quarks

gluons

valence quarks



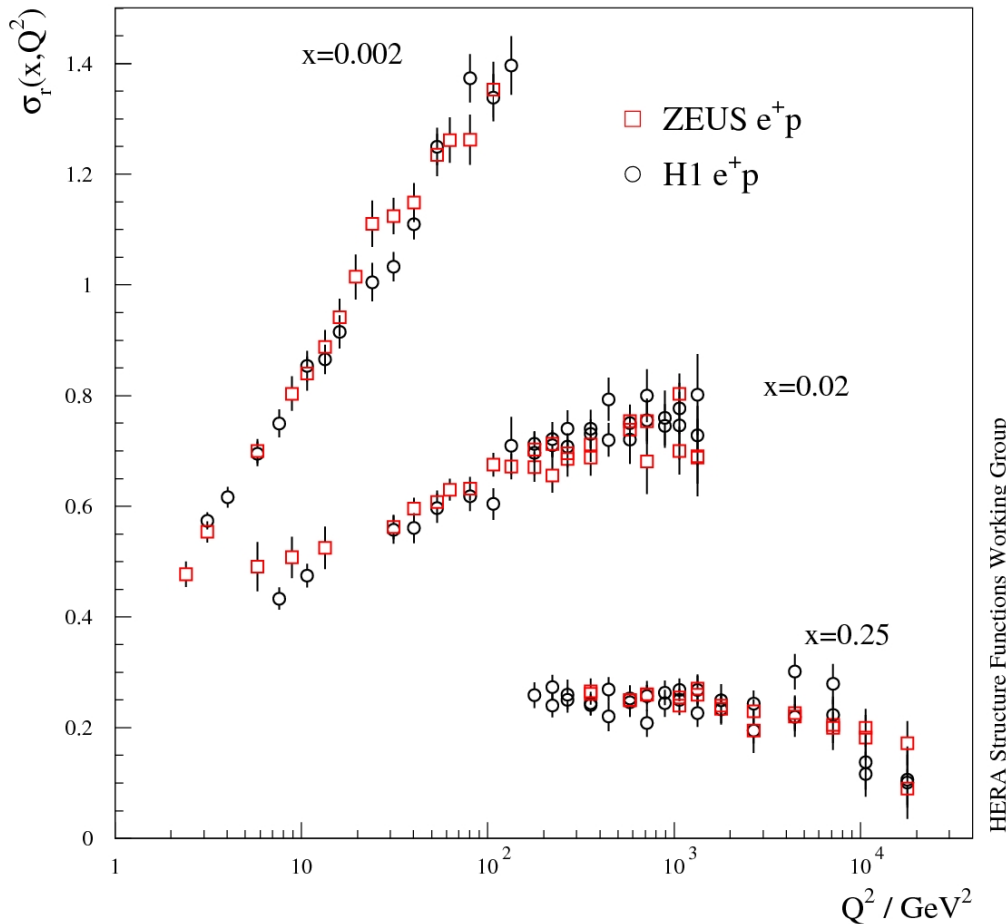
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Example NC cross sections

$$\sigma_r^{e^+p \rightarrow e^+X} = F_2 - \frac{y^2}{Y_+} F_L \mp \frac{Y_-}{Y_+} x F_3$$

HERA I e^+p Neutral Current Scattering – H1 and ZEUS



- Precise measurements from two experiments

- For $Q^2 \leq 100 \text{ GeV}^2$

$$\delta_{\text{stat}} \leq 1\%, \delta_{\text{sys}} \leq 3\%$$

for $Q^2 \geq 1000 \text{ GeV}^2$

$$\delta_{\text{stat}} > \delta_{\text{sys}}$$

- Combine dataset from both experiments:

Key assumption

H1 and ZEUS measure the same cross section at the same x, Q^2, y

- Model independent combination by minimizing a $\chi^2 \rightarrow$



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χ^2 Definition

$$\chi_{\text{exp}}^2(M^{i,\text{true}}, \Delta\alpha_j) = \sum_i \frac{\left[M^{i,\text{true}} - \left(M^i + \sum_j \frac{\partial M^i}{\partial \alpha_j} \Delta\alpha_j \right) \right]^2}{\sigma_i^2} + \sum_j \frac{\Delta\alpha_j^2}{\sigma_{\alpha_j}^2}$$

M^i measured central values

σ_i statistical and uncorrelated systematic uncertainties

σ_{α_j} correlated uncertainty

$\frac{\partial M^i}{\partial \alpha_j}$ Sensitivity of the data to the systematic source j

$M^{i,\text{true}}$ Fitted H1-ZEUS combined cross section

$\frac{\partial M^i}{\partial \alpha_j} \Delta\alpha_j$ Fitted shift of the /data due to the systematic source j

If $\Delta\alpha_j = 0$ it coincides with a standard average

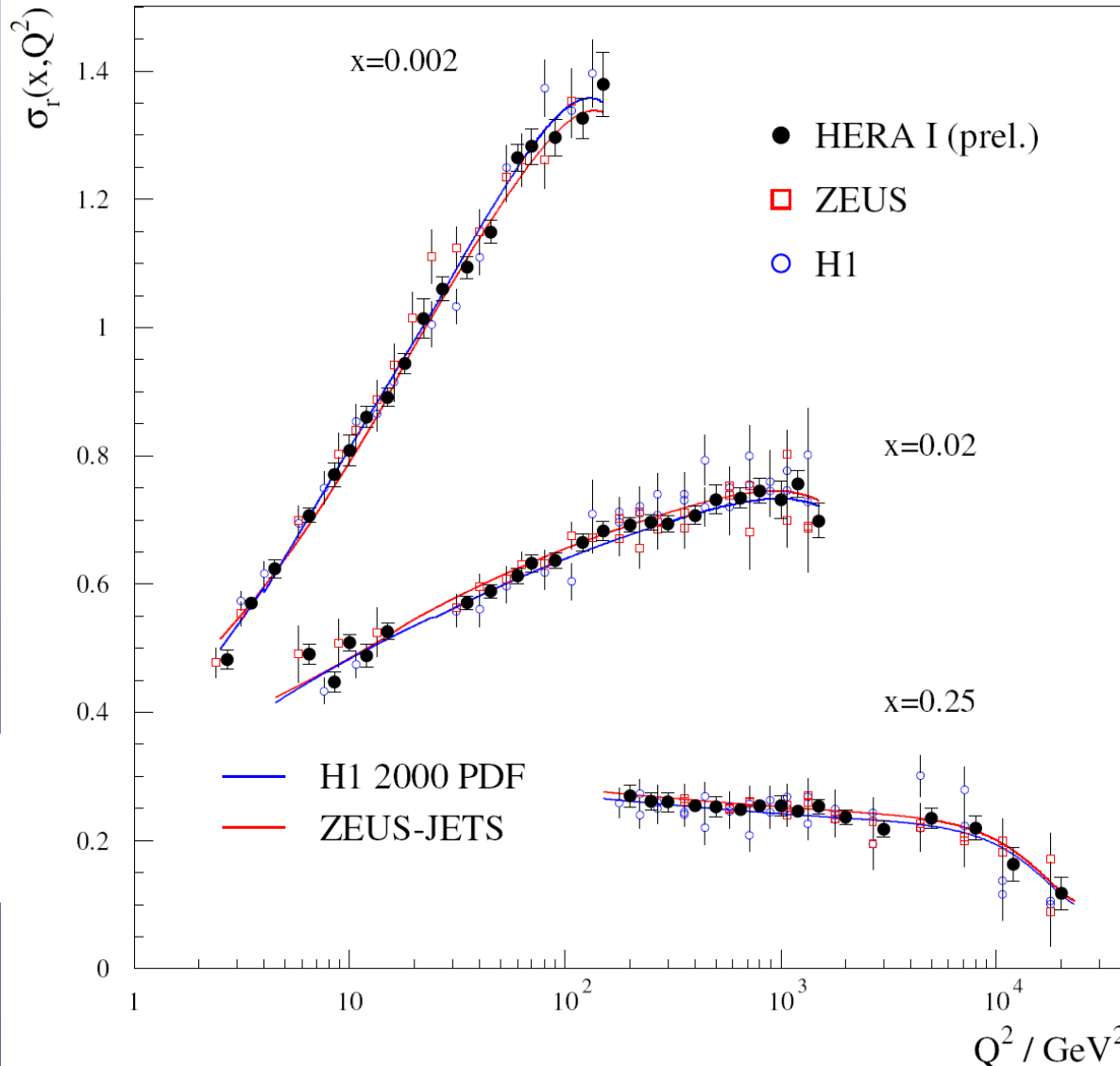
Caution: Most errors are provided as relative errors, a smaller value of cross section has smaller absolute error bias toward smaller averages

Can be avoided by modified χ^2 definition: insert $M^{i,\text{true}}/M^i$



Averaged NC cross sections

HERA I e^+p Neutral Current Scattering - H1 and ZEUS



Improved precision of combined H1 and ZEUS datasets

Total uncertainty includes ambiguity whether to use systematic errors as additive or multiplicative errors

Here predictions use PDFs of H1 and ZEUS separate NLO QCD PDF extractions.

Next step H1 and ZEUS combined fit

HERA Structure Functions Working Group

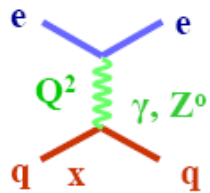


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Input to HERA PDF Fits

■ NC
$$\frac{d^2 \sigma^{e^\pm p \rightarrow e^\pm X}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \underbrace{\left(1 + (1-y)^2\right)}_{Y_\pm = 1 \pm (1-y)^2} \cdot \left(\tilde{F}_2(x, Q^2) - \frac{y^2}{Y_+} \tilde{F}_L(x, Q^2) \mp \frac{Y_-}{Y_+} x\tilde{F}_3(x, Q^2) \right)$$

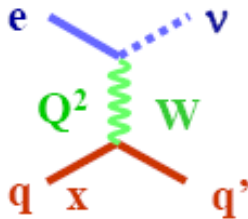


$$\tilde{F}_2 = \sum_i A_i(Q^2) [xq_i + x\bar{q}_i] \Rightarrow F_2^{em} = \frac{4}{9}x(u + \bar{u} + c + \bar{c}) + \frac{1}{9}x(d + \bar{d} + s + \bar{s})$$

$$x\tilde{F}_3 = \sum_i B_i(Q^2) [xq_i - x\bar{q}_i] \Rightarrow xF_3^{em} = B_U x(u - \bar{u} + c - \bar{c}) + B_D x(d - \bar{d} + s - \bar{s})$$

Electroweak Coefficient Functions $A_i(Q^2)$, $B_i(Q^2)$ (QED: $A_i = e_i^2$)

■ CC



$$\sigma_{CC}(e^+ p) \propto x \left[(1-y^2)(d+s) + (\bar{u} + \bar{c}) \right] \times (1 + P_e)$$

$$\sigma_{CC}(e^- p) \propto x \left[(u+c) + (1-y^2)(\bar{d} + \bar{s}) \right] \times (1 - P_e)$$

PDF general form: $xPDF = Ax^B (1-x)^C \cdot (1 + Dx + \dots)$

Parameterize: g , u_v , d_v , $U(=u+c)$, $D(=d+s)$

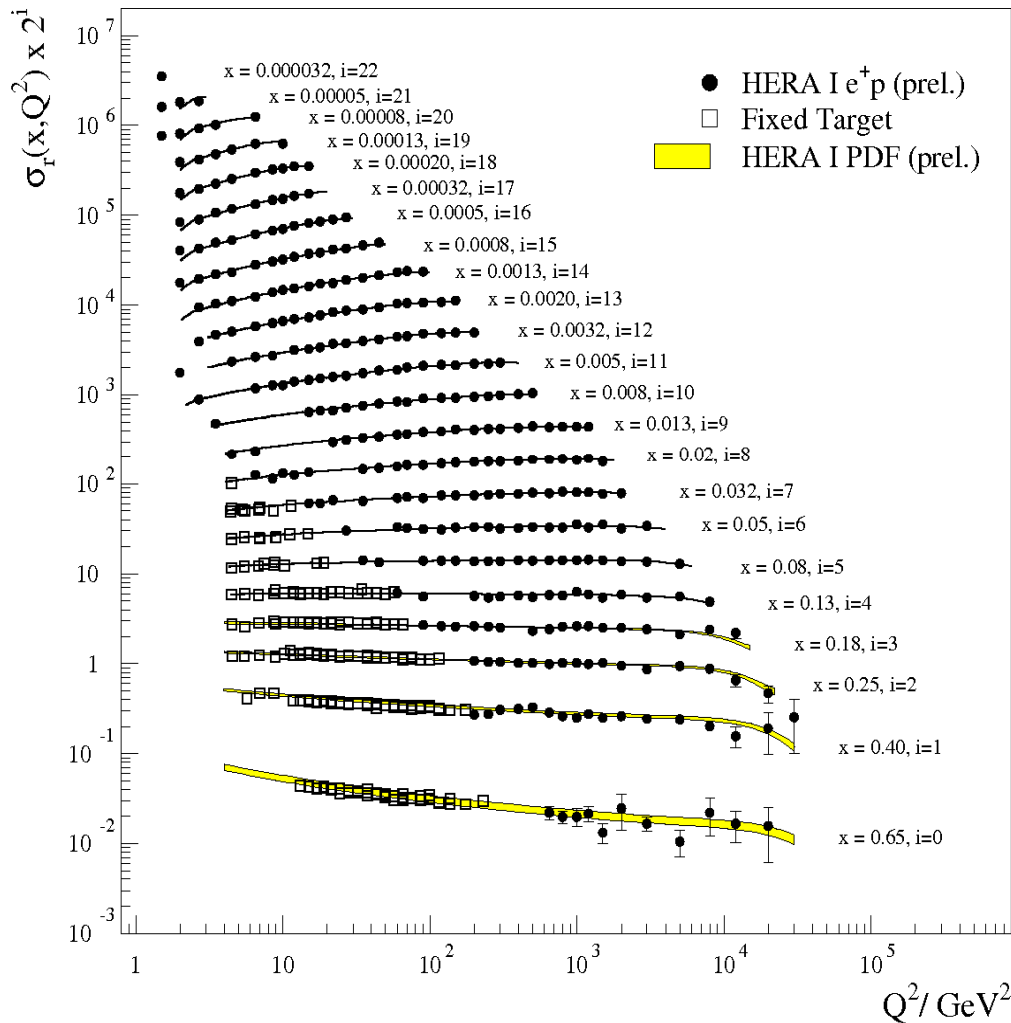


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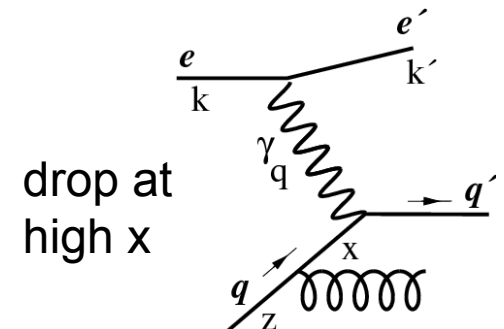
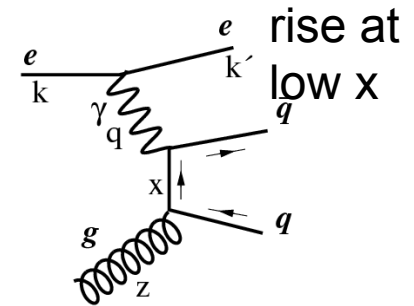
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Structure Function F_2

H1 and ZEUS Combined PDF Fit



- H1 & ZEUS extended fixed target kinematic regime in x and Q^2 by 2 Orders
- Described by DGLAP
- Scaling violations

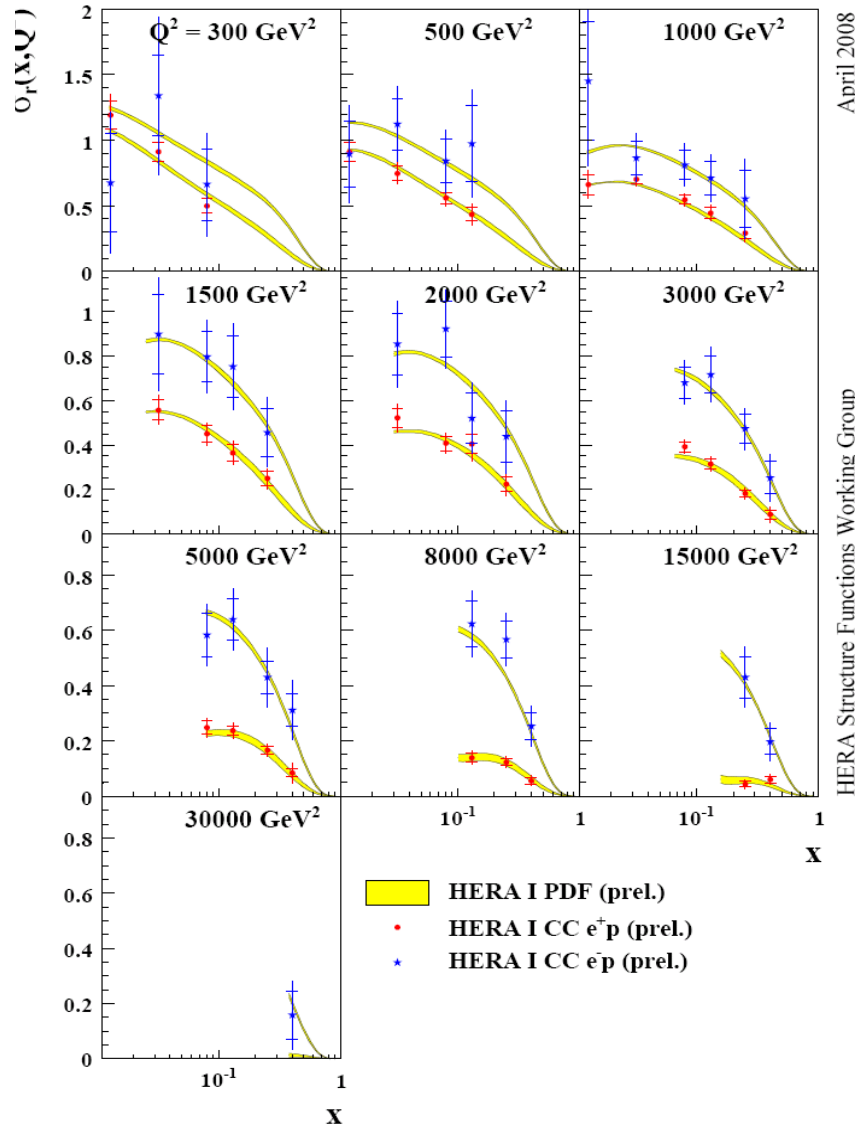


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Charged Current Cross Section

H1 and ZEUS Combined PDF Fit



CC Cross section provide flavor sensible constraints at high x

$$\sigma_{CC}(e^+p) \propto x \left[(1-y^2)D + \bar{U} \right]$$

$$\sigma_{CC}(e^-p) \propto x \left[U + (1-y^2)\bar{D} \right]$$

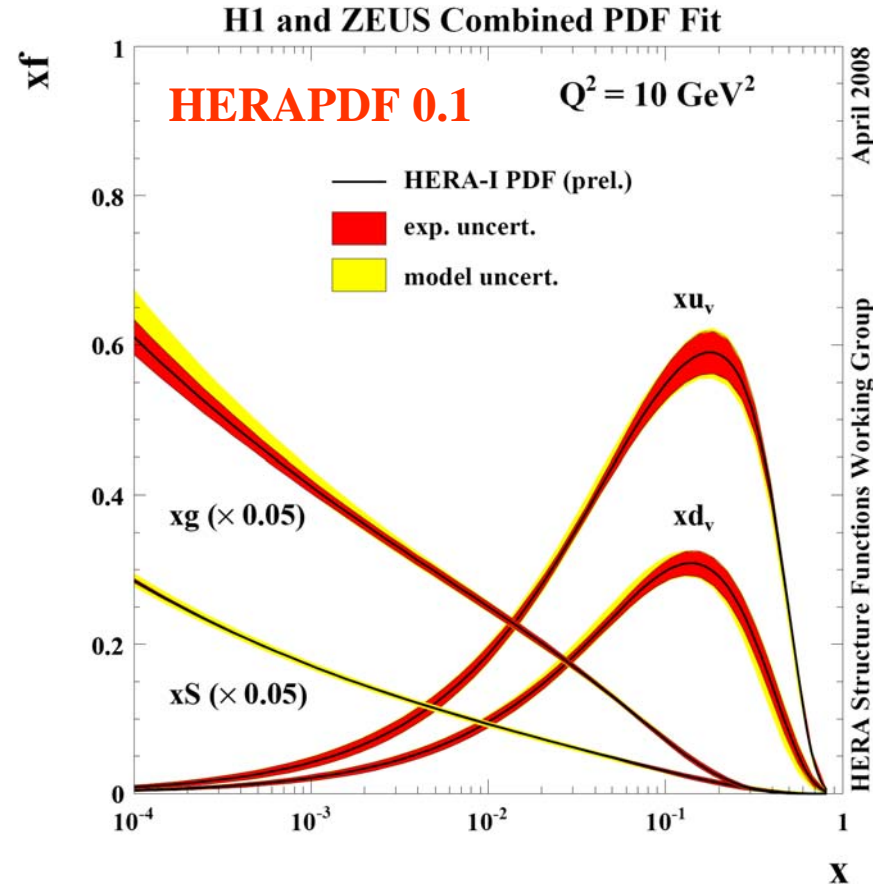
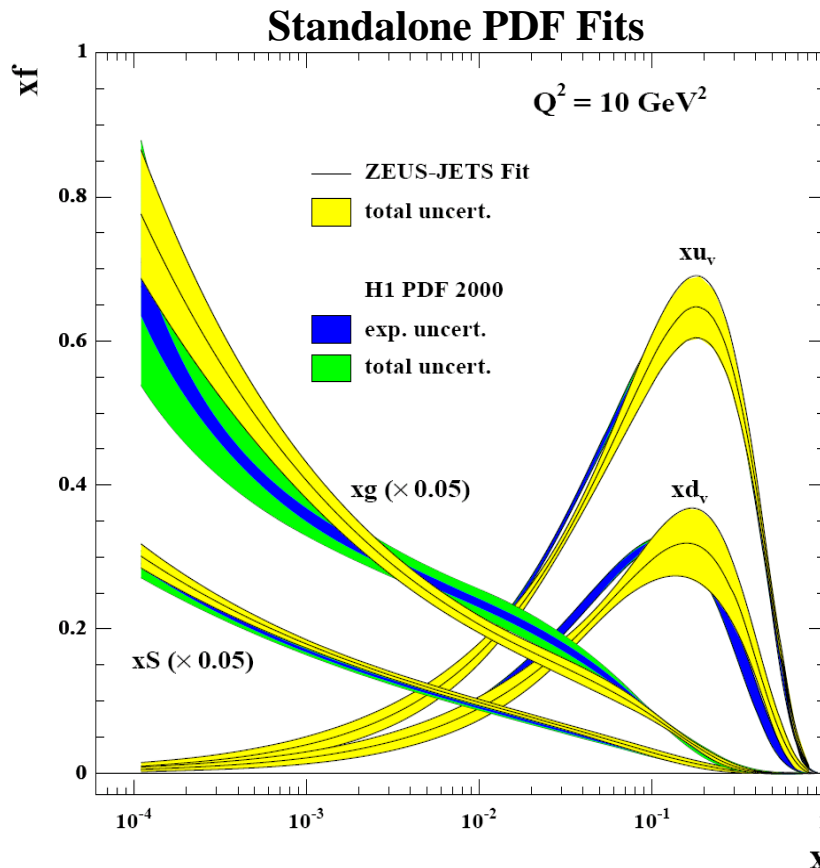
Improved precision of σ_{CC}
By combining H1 and ZEUS



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PDF Fits on HERA I Data



Impressive reduction of uncertainties of combined

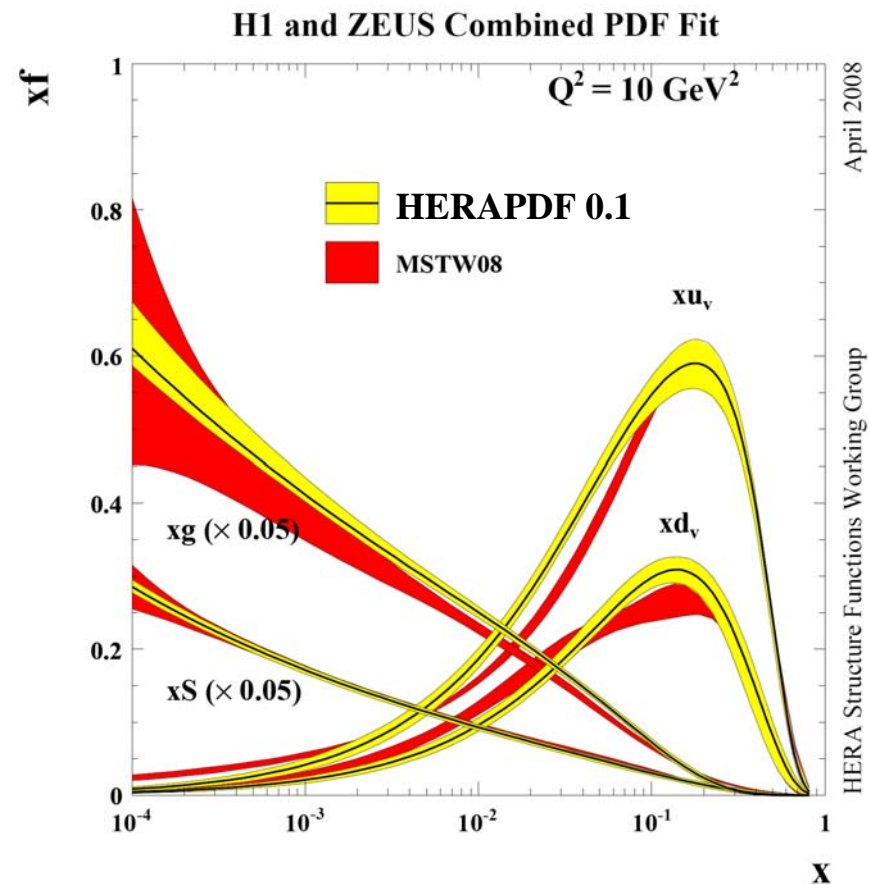
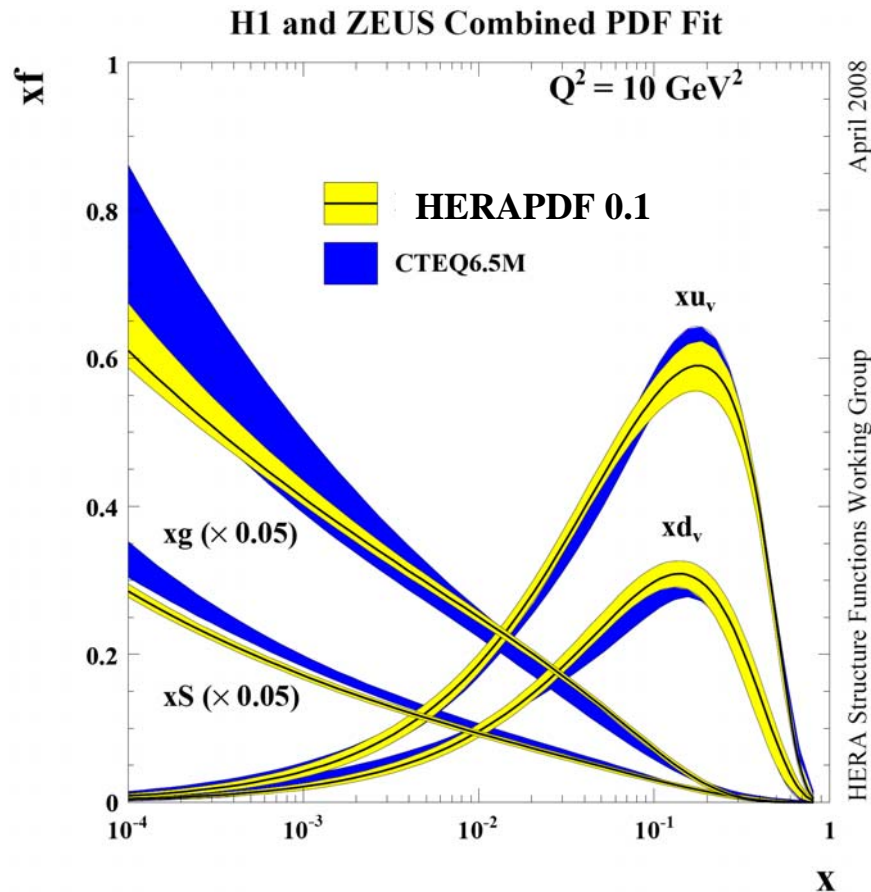
Model uncertainty: variation of charm and bottom mass, starting scale Q_0^2 , Q_{\min}^2 of included data, strange and charm fraction at starting scale



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Comparison to Global Fits

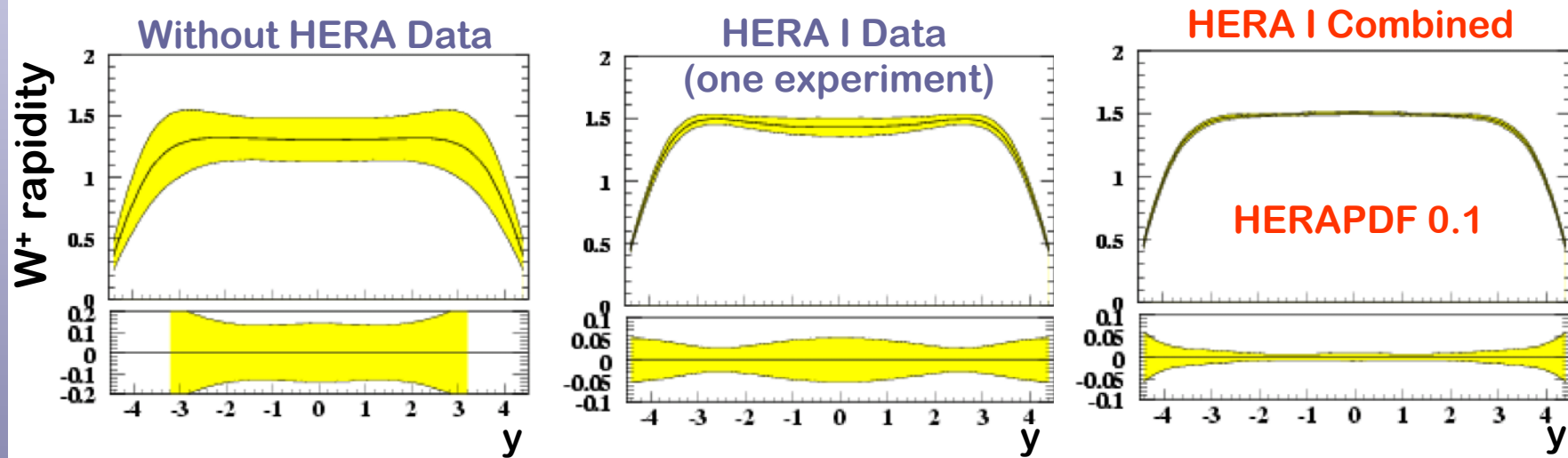


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Impact of HERA PDFs

Example: W^+ production at the LHC (Study by A. Cooper-Sakar)



- HERA Combined Data and PDFs are an crucial input to predictions at the LHC
- Note: Error bands are experimental uncertainties only
model uncertainty will become increasingly important

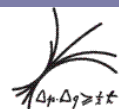


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Summary

- Combining H1 and ZEUS reduces statistical and systematic uncertainties
- Model independent combination procedure
- HERA PDF fit on combined HERA I data
 - use ep data only → no need for nuclear corrections
 - results in a HERA PDF set with impressive precision compared to previous HERA analyses, and to the global fits
- HERAPDF0.1 is being implemented in LHAPDF
- Still to come: Use all of HERA II data



Backup Slides

- Data combination: Modified χ^2
- HERAPDF0.1 Parameterization
- Comparison of H1 and ZEUS PDF fits
- Model uncertainties



Modified c2 definition

Caution: Most errors are provided as relative errors, a smaller value of cross section has smaller absolute error bias toward smaller averages
Can be avoided by modified χ^2 definition:

$$\chi_{\text{exp}}^2 \left(M^{i,\text{true}}, \Delta\alpha_j \right) = \sum_i \frac{\left[M^{i,\text{true}} - \left(M^i + \sum_j \frac{\partial M^i}{\partial \alpha_j} \frac{M^{i,\text{true}}}{M^i} \Delta\alpha_j \right) \right]^2}{\left(\sigma_i \frac{M^{i,\text{true}}}{M^i} \right)^2} + \sum_j \frac{\Delta\alpha_j^2}{\sigma_{\alpha_j}^2}$$

Normalization uncertainty is clearly a relative (multiplicative).

Are other systematic errors as additive or multiplicative errors?

Choice of best treatment is debatable. Does it matter?

Impact is mostly negligible, except at very large Q2 and x where statistical errors and fluctuation are largest.

How to deal with this freedom in systematic error treatment?












→ Additional correlated uncertainty of averaged data points



Details on HERAPDF 0.1

Chosen form of the PDF parametrization at Q_0^2

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2+Fx^3\dots)$$

	A	B	C	D	E
gluon	sum rule				
u_v	sum rule				
d_v	sum rule	$= B(u_v)$			
U_{bar}	Lim $x \rightarrow 0 \ u/d \rightarrow 1$				
D_{bar}		$= B(U)$			

The number of parameters for each parton has been optimized

Optimization means starting with only **BLUE** parameters and **adding D, E, F** parameters until there is no further χ^2 advantage

PDFs fitted: gluon, u_v , d_v , $U_{bar} = u_{bar} + c_{bar}$, $D_{bar} = d_{bar} + s_{bar} + b_{bar}$

Sea flavour break-up at Q_0 : $s = fs \cdot D$, $c = fc \cdot U$, $AU_{bar} = (1-fs)/(1-fc)AD_{bar}$

Lim $x \rightarrow 0 \ u_{bar}/d_{bar} \rightarrow 1$

$fs = 0.33D$ ($s=0.5d$), $fc = 0.15U$ **consistent with dynamical generation**

$m_c = 1.4 \text{ GeV}$ **mass of charm quark** $m_b = 4.75 \text{ GeV}$ **mass of beauty quark**

Zero-mass variable flavour number heavy quark scheme (for now)

$Q_0^2 = 4 \text{ GeV}^2$ **input scale**

$Q_{min}^2 = 3.5 \text{ GeV}^2$ **minimum Q^2 of input data**

$\alpha_s(M_Z) = 0.1176$ **PDG2006 value**

Renormalization and factorization scales = Q^2



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Comparison of Separate H1 and ZEUS PDF Fits

$$xf(x) = Ax^B(1-x)^C(1 + Dx + Ex^2 + Fx^3 \dots)$$

Alternative form of PDF parametrization: H1 style

	A	B	C	D	E	F
gluon	sum rule					
U	$\lim_{x \rightarrow 0} \bar{u}/\bar{d} \rightarrow 1$			sum rule		
D		$= B(U)$		sum rule		
U_{bar}	$= A(U)$	$= B(U)$				
D_{bar}	$= A(D)$	$= B(U)$				

PDFs: gluon, $U=u+c$, $U_{\text{bar}}=u_{\text{bar}}+c_{\text{bar}}$, $D=d+s+b$, $D_{\text{bar}}=d_{\text{bar}}+s_{\text{bar}}+b_{\text{bar}}$

Sea flavour break-up at Q_0 : $s = f_s \cdot D$, $c = f_c \cdot U$ $AU = (1-f_s)/(1-f_c)AD$

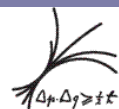
Alternative form of PDF parametrization: ZEUS style

	A	B	C	D	E
gluon	From Sum Rule				0.
u_v	From Sum Rule				
d_v	From Sum Rule	$= B_{uv}$			0.
$u_{\text{bar}} - d_{\text{bar}}$	from Z S 11 fit	from Z S 11 fit	from Z S 11 fit	0.	0.
Sea				0.	0.

PDFs: gluon, u_v , d_v , Sea = $u_{\text{sea}} + u_{\text{bar}} + d_{\text{sea}} + d_{\text{bar}} + s + s_{\text{bar}} + c + c_{\text{bar}}$

Sea flavour break-up at Q_0 : $s_{\text{bar}} = (d_{\text{bar}} + u_{\text{bar}})/4$, charm dynamically generated,

$d_{\text{bar}} - u_{\text{bar}}$ fixed to fit E866 data



Model Uncertainties

	Model Variation	Standard value	Upper limit	Lower limit
m_c	Mass c quark	1.4	1.35	1.5
m_b	B quark mass	4.7	4.3	5.0
f_s	Strange sea frac. (D) at Q_0^2	0.33	0.25	0.40
f_c	Charm sea frac. (U) at Q_0^2	0.15	0.12	0.18
Q_0^2	Starting scale	4.0	2.0	6.0
Q_{\min}^2	cut of included data	3.5	2.5	5.0

- Correlated variations
 f_c varies when m_c is varied
variation Q_0^2 also changes f_s and f_c

