H1 and ZEUS Combined PDF Fit

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Outline: NLO DGLAP PDF fit to the combined HERA data set

- Data sets
- Choice of parametrization
- Choice of error treatment
- Model assumptions
- Results
  - Quality of fit to data
  - PDFs
  - Comparisons to older H1/ZEUS fits, and to CTEQ, MRST
  - Model Variations
- summary

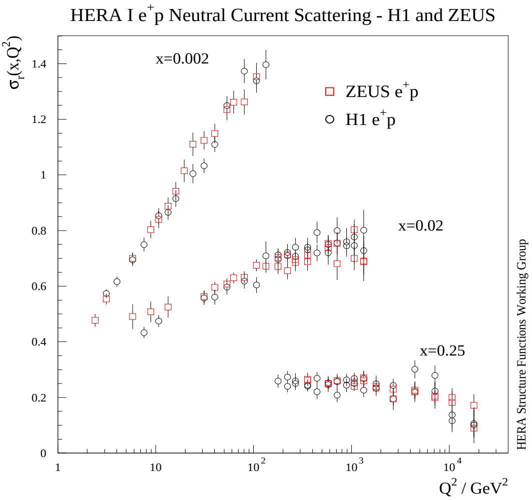
Data sets

Based on published data sets of HERA I($\sim 115pb^{-1}$ per experiment)

data set		x range		Q^2 range(GeV^2)		$\mathcal{L}(pb^{-1})$	comment
H1 NC min. bias	97	0.00008	0.02	1.5	12	1.8	$e^+p\sqrt{s} = 301$GeV
H1 NC Low Q^2	96—97	0.000161	0.20	12	150	17.9	$e^+p\sqrt{s} = 301$GeV
H1 NC	94—97	0.0032	0.65	150	30000	35.6	$e^+p\sqrt{s} = 301$GeV
H1 CC	94—97	0.013	0.40	300	15000	35.6	$e^+p\sqrt{s} = 301$GeV
H1 NC	98—99	0.0032	0.65	150	30000	16.4	$e^-p\sqrt{s} = 319$GeV
H1 CC	98—99	0.0013	0.40	300	15000	16.4	$e^-p\sqrt{s} = 319$GeV
H1 NC	99—00	0.00131	0.65	100	30000	65.2	$e^+p\sqrt{s} = 319$GeV
H1 CC	99—00	0.013	0.40	300	15000	65.2	$e^+p\sqrt{s} = 319$GeV
ZEUS NC	96—97	0.00006	0.65	2.7	30000	30.0	$e^+p\sqrt{s} = 301$GeV
ZEUS CC	94—97	0.015	0.42	280	17000	47.7	$e^+p\sqrt{s} = 301$GeV
ZEUS NC	98—99	0.005	0.65	200	30000	15.9	$e^-p\sqrt{s} = 319$GeV
ZEUS CC	98—99	0.015	0.42	280	30000	16.4	$e^-p\sqrt{s} = 319$GeV
ZEUS NC	99—00	0.005	0.65	200	30000	63.2	$e^+p\sqrt{s} = 319$GeV
ZEUS CC	99—00	0.008	0.42	280	17000	60.9	$e^+p\sqrt{s} = 319$GeV

With H1 NC min. bias ($Q^2 < 12$GeV^2) moved up by 3.4% after reanalysis of luminosity.

Data Sets: Example of NC cross section



Precise measurements from two experiments

For $Q^2 \leq 100\text{GeV}^2$, $\delta_{stat} \leq 1\%$, $\delta_{sys} \leq 3\%$;

For $Q^2 \geq 1000\text{GeV}^2$,

$\delta_{stat} > \delta_{sys}$

Data sets: Combination procedure

1. Move all data points to a common $x - Q^2$ grid
   - Grid: H1 x binning and ZEUS Q^2 binning basically
   - Straightforward interpolation:

$$\sigma_{ep}^{meas}(x_{grid}, Q^2_{grid}) = \frac{\sigma_{ep}^{th}(x_{grid}, Q^2_{grid})}{\sigma_{ep}^{th}(x, Q^2)} \sigma_{ep}^{meas}(x, Q^2)$$

   - H1PDF2k and ZEUS-Jets fits have been used
2. Move 820 GeV data 920 GeV beam energy
   - CC:

$$\sigma_{CC}^{e^{\pm}p}{}_{920}(x, Q^2) = \sigma_{CC}^{e^{\pm}p}{}_{820}(x, Q^2) \frac{\sigma_{CC}^{th,e^{\pm}p}{}_{920}(x, Q^2)}{\sigma_{CC}^{th,e^{\pm}p}{}_{820}(x, Q^2)}$$

   - NC:

$$\sigma_{NC}^{e^{\pm}p}{}_{920}(x, Q^2) = \sigma_{NC}^{e^{\pm}p}{}_{820}(x, Q^2) + \Delta\sigma_{NC}^{e^{\pm}p}(x, Q^2, y_{920}, y_{820})$$

3. Calculate the average values, errors and the uncertainties related to the combination method

Data sets: Averaging Mehtod

- Model independent combination, the key assumption is that H1 and ZEUS experiments are measuring the same cross sections at the same kinematical points. See A. Glazov DIS05
- It minimises the χ^2

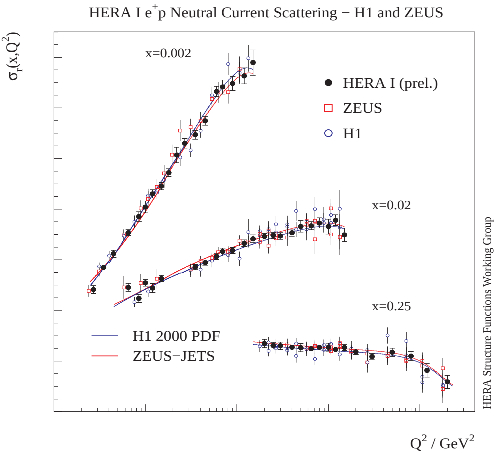
$$\chi^2_{exp}(M^{i,true}, \Delta\alpha_j) = \sum_i \frac{\left[M^{i,true} - \left(M^i + \sum_j \frac{\partial M^i}{\partial \alpha_j}\Delta\alpha_j\right)\right]^2}{\sigma_i^2} + \sum_j \frac{\Delta\alpha_j^2}{\sigma_{\alpha_j}^2}$$

  - $M^{i,true}$: measured central values
  - σ_i^2: statistical and uncorrelated systematic uncertainties
  - σ_{α_j}: correlated uncertainty
  - $\frac{\partial M^i}{\partial \alpha_j}$: Sensitivity of the data to the systematic source j
  - M^i: Fitted H1-ZEUS combined cross section
  - $\frac{\partial M^i}{\partial \alpha_j}\Delta\alpha_j$: Fitted shift of the I data due to the systematic source j

  If $\Delta\alpha_j = 0$, it coincides with a standard average

Caution: Most errors are provided as relative errors, a smaller value of cross section has smaller absolute error bias toward smaller averages Can be avoided by modified χ^2 definition: insert $\frac{M^{i,true}}{M^i}$

Data Sets: Averaged NC cross sections



Improved precision of combined H1 and ZEUS data sets

Here predictions use PDFs of H1 and ZEUS separate NLO QCD PDF extractions.

Next step: H1 and ZEUS combined fit

Chosen form of the PDF parameterization at Q_0^2

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2+Fx^3+\ldots)$$

	A	B	C	D	E
gluon	sum rule	■	■		
u_v	sum rule	■	■	■	■
d_v	sum rule	$= B(u_v)$	■		
$\bar{U}$	$\lim_{x\to 0} \bar{U}/\bar{D} \to 1$	■	■		
$\bar{D}$	■	$= B(\bar{U})$	■		

- The number of parameters for each parton has been optimized
- Optimization means starting with only BLUE parameters and adding D, E,F parameters until there is no further χ^2 advantage
- PDFs fitted gluon, u_v, d_v, $\bar{U} = \bar{u} + \bar{c}$, $\bar{D} = \bar{d} + \bar{s}$

  Sea flavour break-up at Q_0: $s = fs \times \bar{D}$, $c = fc \times \bar{U}$, $A_{\bar{U}} = (1-fs)/(1-fc)A_{\bar{D}}$

  fs=0.33, fc=0.15: consistent with dynamical generation
- m_c=1.4GeV (mass of charm quark) m_b=4.75GeV (mass of beauty quark)
- Zero-mass variable flavour number schmeme heavy quark scheme(for now)
- $Q_0^2 = 4$ GeV^2 input scale $\alpha_s(M_Z) = 0.1176$ PDG2006 value
- Renormalization and factorizatin scale $= Q^2$

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2+Fx^3+\ldots)$$

- Alternative form of PDF parameterization: H1 Style

	A	B	C	D	E	F
gluon	sum rule					
U	$\lim_{x\to 0} \bar{u}/\bar{d} \to 1$			sum rule		
D		$= B(U)$		sum rule		
U_{bar}	$= A(U)$	$= B(U)$				
D_{bar}	$= A(D)$	$= B(U)$				

PDFs fitted gluon, $U = u + c$, $D = d + s$, $\bar{U} = \bar{u} + \bar{c}$, $\bar{D} = \bar{d} + \bar{s}$

Sea flavour break-up at Q_0: $s = fs \times \bar{D}$, $c = fc \times \bar{U}$, $A_{\bar{U}} = (1-fs)/(1-fc)A_{\bar{D}}$

- Alternative form of PDF parameterization: ZEUS Style

	A	B	C	D	E	F
gluon	sum rule					
u_v	sum rule					
d_v	sum rule	$= B_{uv}$		sum rule		
$\bar{u}-\bar{d}$	From Z_S_11_fit					
Sea						

PDFs fitted gluon, u_v, d_v, $Sea = u_{sea} + \bar{u} + d_{sea} + \bar{d} + s + \bar{s} + c + \bar{c}$,

Sea flavour break-up at Q_0: $\bar{s} = (\bar{d} + \bar{u})/4$, charm dynamically generated,

$\bar{u} - \bar{d}$ fixed to fit E866 data

Choice of parameterization

All three forms have good χ^2, our choice has the best

Further motivations are:

- Less Model dependence on B paramters than H1 parameterization.
- No need for an additional input $(\bar{d} - \bar{u})$ x-distribubtion in ZEUS parameterization.
- Most conservative errors.
- It is inspired by both H1 and ZEUS parameterizations.

Choice of experimental error treatment

- The data have already been combined taking full account of their correlated systematic errors, resulting in much reduced systematic uncertainties on the combined data set.
  Systematic uncertainties are now smaller than statistical uncertainties across the x, Q^2 plane.
- We combine the 43 systematic uncertainties of the data with the statistical uncertainties in quadrature. Then we OFFSET the 4 systematic uncertainties which result from the combination procedure:
  $$\chi^2 = 476.7 \text{ for 562 degrees of freedom.}$$
  For comparison:
  treating all 47 systematic sources quadratically gives $\chi^2 = 428.0$
  treating all 47 systematic sources as still correlated gives $\chi^2 = 553.1$
- All three methods give very similar central values for PDFs and very similar PDF uncertainties. Our choice is the most conservative.

The self-consistency of our data set and small systematics allows us to use $\Delta\chi^2 = 1$ to calculate the uncertainties.

Model uncertainties: to be added into the total PDF uncertainty

- m_c : 1.3 $\rightarrow$ 1.55GeV variation of mass of c quark
- m_b : 4.3 $\rightarrow$ 5.0 GeV variation of mass of b quark
- fs : 0.25 $\rightarrow$ 0.40 variation of strange sea fraction at Q_0^2
- fc : 0.10 $\rightarrow$ 0.20 variation of charm sea fraction at Q_0^2
- Q_0^2 : 2.0 $\rightarrow$ 6.0 GeV^2 variation of starting scale
- Q^2_{min} : 2.5 $\rightarrow$ 5.0 GeV^2 variation of cuts on the data included

Correlated variations: f_c varies when is m_c is varied, variation Q_0^2 also changes f_s and f_c.

Model variations: to be compared with our results

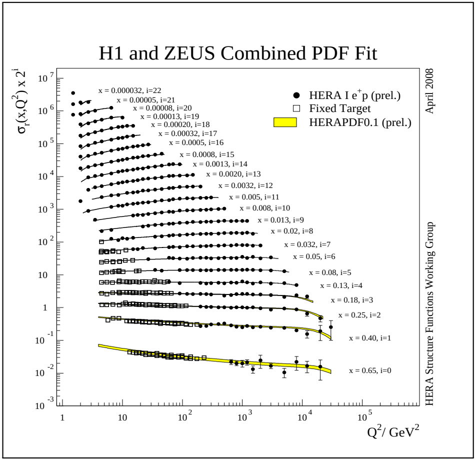
- Variation of $\alpha_s(M_z)$: 0.1156 $\rightarrow$ 0.1196
- Variation of form of parametrization

Fit Results

PDF fit RESULTS

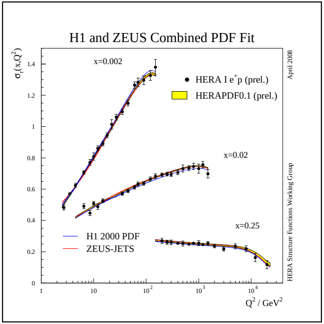
Comparison to HERA combined data

Quality of fit to data



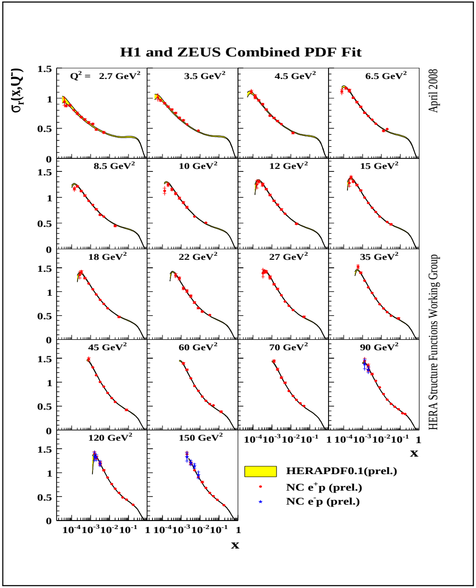
New HERA-I PDF fit predictions vs. H1/ZEUS combined data for NC e^+p.

Total uncertainties on the PDF fit predictions are included but can barely be resolved.



Blow up just three x values to see older ZEUS-JETS PDF and H1PDF2000 plus new **HERAPDF0.1**

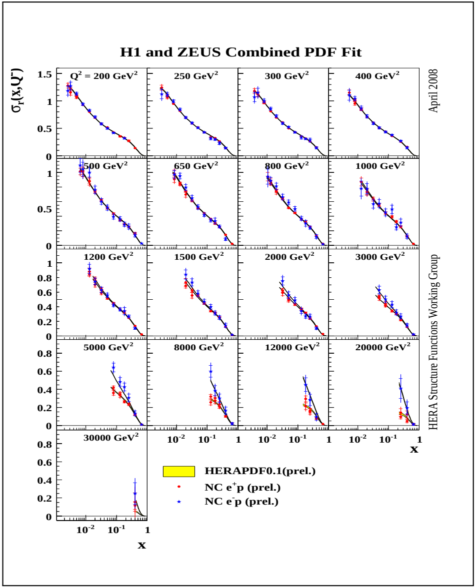
Quality of fit to data



New H1/ZEUS combined PDF fit predictions vs. H1/ZEUS combined data for NC e^+p and e^-p at low Q^2.

Total uncertainties on the PDF fit predictions are included but barely be resolved.

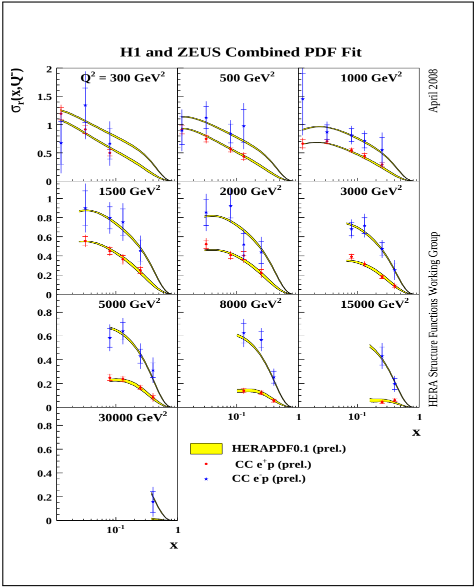
Quality of fit to data



New H1/ZEUS combined PDF fit predictions vs. H1/ZEUS combined data for NC e^+p and e^-p at high Q^2.

Total uncertainties on the PDF fit predictions are included but cannot be resolved.

Quality of fit to data



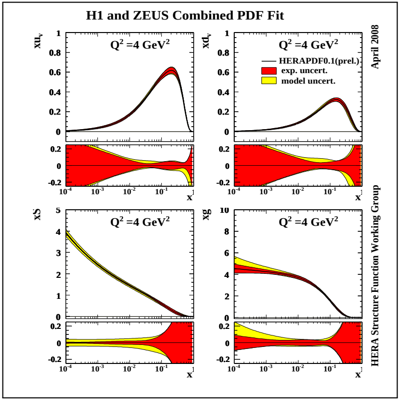
New H1/ZEUS combined PDF fit predictions vs. H1/ZEUS combined data for CC e^+p and e^-p at high Q^2.

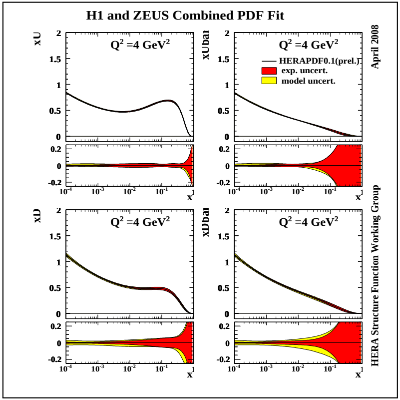
Total uncertainties on the PDF fit predictions are included.

PDFs

HERAPDF0.1
PDFs: **experimental** and **model** errors
Comparison to other PDFs

At the starting scale: $Q_0^2 = 4\text{GeV}^2$



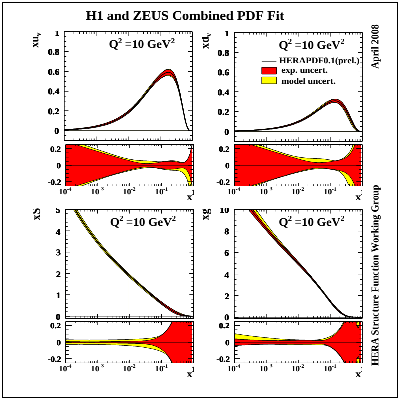


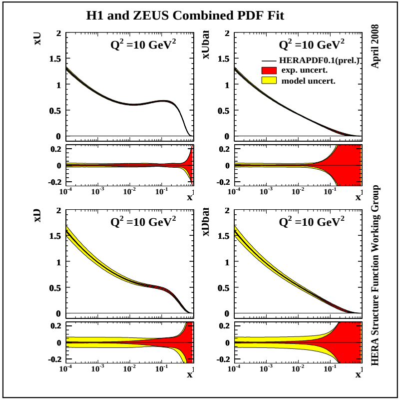
New H1/ZEUS combined PDFs with **total experimental uncertainty bands** plus **model uncertainty bands**

Strange fraction is the major contribution to model uncertainty on the sea

Choice of starting scale and Q^2 cuts to the valence and gluon

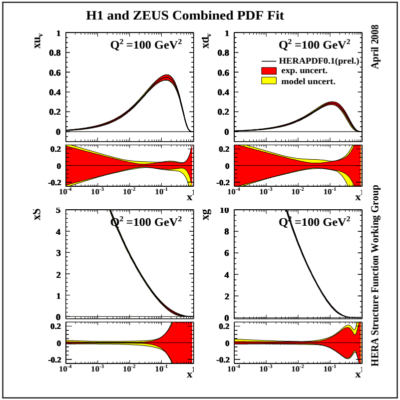
At $Q^2 = 10\text{GeV}^2$

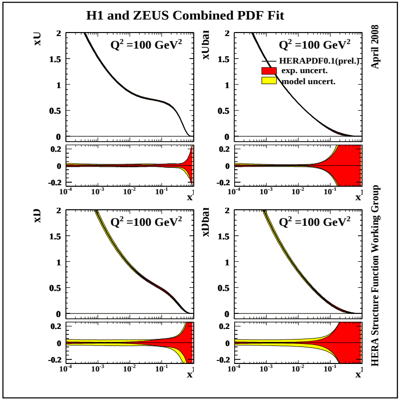




New H1/ZEUS combined PDFs with **total experimental uncertainty bands** plus **model uncertainty bands**

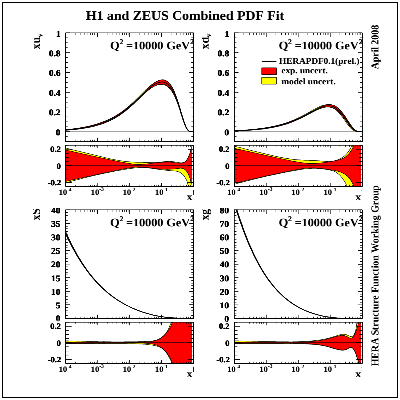
At $Q^2 = 100\text{GeV}^2$

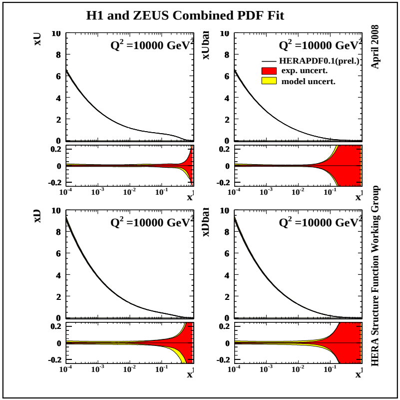




New H1/ZEUS combined PDFs with **total experimental uncertainty bands** plus **model uncertainty bands** from 6 sources of model variation:
Note how uncertainties are decreasing with Q^2

At $Q^2 = 10000\text{GeV}^2$

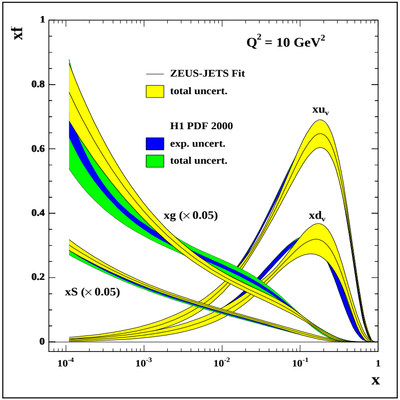


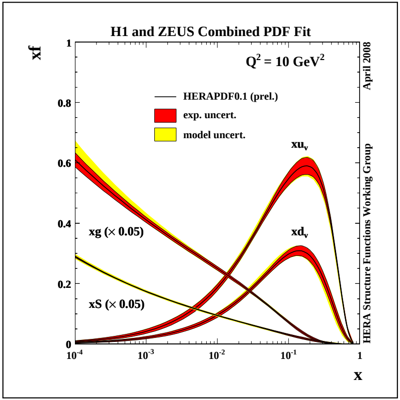


New H1/ZEUS combined PDFs with **total experimental uncertainty bands** plus **model uncertainty bands** from 6 sources of model variation:
At scales relevant to LHC physics uncertainties are impressively small.

Compare to published ZEUS/H1 results which also used only HERA data

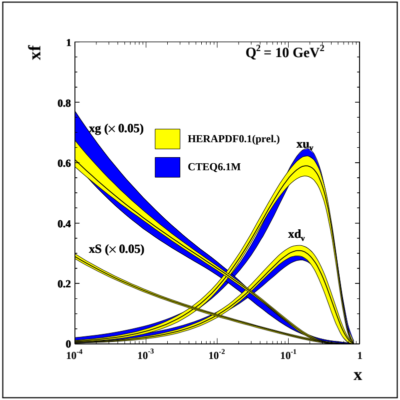
Note in published PDFs H1 has α_s variation included in model error, ZEUS does not.

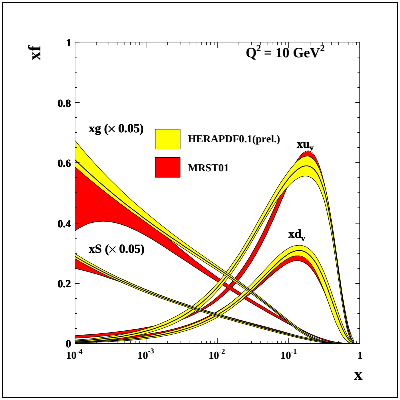




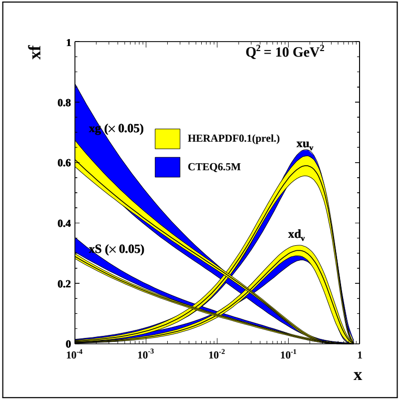
Resolution of previous discrepancies, improvement in level of uncertainty

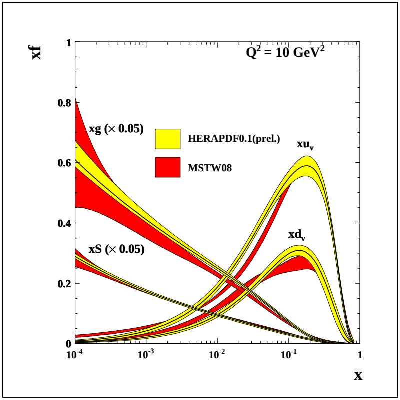
Compare to CTEQ and MRST analyses: older



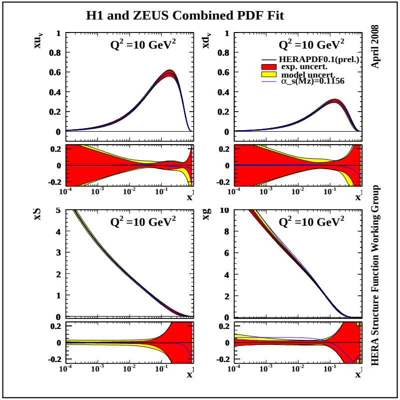


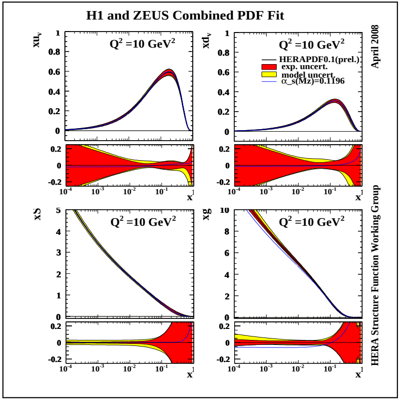
Compare to CTEQ and MRST(prel.): newer





Variations: α_s

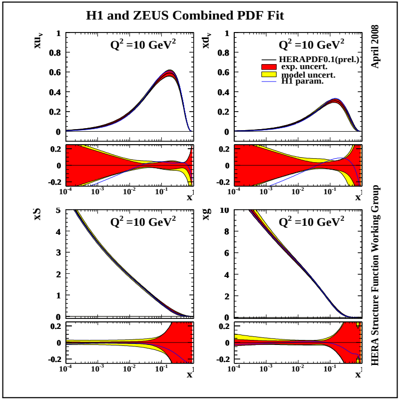


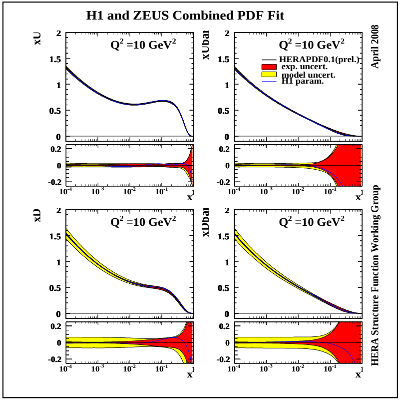


Comparison of central fit plus **total uncertainties** to variations with $\alpha_s(M_Z) = 0.1156$ **(left) and 0.1196 (right)**

Variation is outside the gluon error bands even when other model dependence is accounted.

Variations: use H1 style parametrization

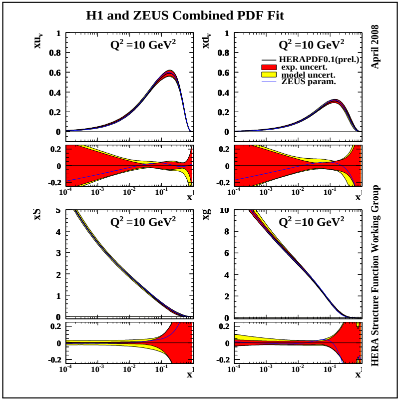


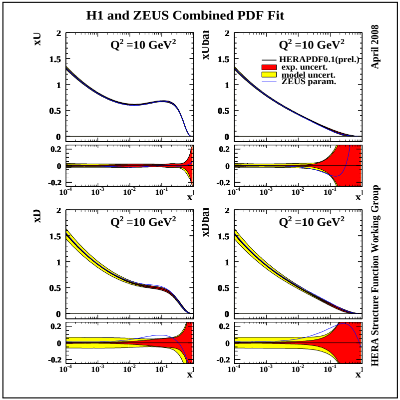


Comparison of central fit plus **total uncertainties** to variations with **New H1 optimised parametrization**

Marginally outside normal error bands for valence even when other model dependence is accounted **(but note this is at low x where valence not very significant)**

Variations: use ZEUS-style parametrization





Comparison of central fit plus **total uncertainties** to variations using
New ZEUS-JETS optimised parametrization
Inside error bands if other model dependence is accounted

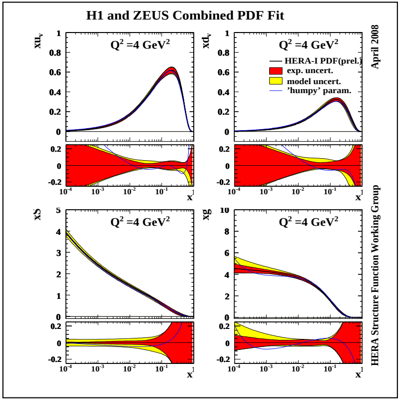
Resolution of an old discrepancy

For each of the parametrizations, if a non-zero D parameter for the gluon is used, there are two minima: straight gluon and humpy gluon solution.

These look rather like the published ZEUS and H1 gluons respectively!

For the H1/ZEUS combined data set the χ^2 of the straight solution is always lower by about 10 χ^2 points. But whereas the humpy solutions are disfavoured by χ^2 they are still acceptable fits.

We compare the humpy and straight solutions for our chosen parametrization here. These parametrizations are very consistent.



Summary

- The combined data set of ZEUS and H1 inclusive cross-section data for NC and CC e^+p and e^-p scattering has greatly improved precision compared to the measurements of either experiment separately
- Differences between ZEUS and H1 PDF fitting analyses have also been resolved. Treatment of experimental and model uncertainties have been carefully considered.
- **Since our combination procedure has resulted in a single data set with consistently treated systematics there is no need for an inflated $\Delta\chi^2$ in setting the errors on the PDFs.**
- **This results in the HERAPDF set which has impressive precision compared to previous HERA analyses, and to the global fits.**

THE END

THANKS A LOT !

Averaging

- Modified χ^2 definition:

$$\chi^2_{exp}(M^{i,true}, \Delta\alpha_j) = \sum_i \frac{\left[M^{i,true} - \left(M^i + \sum_j \frac{\partial M^i}{\partial \alpha_j} \frac{M^{i,true}}{M^i} \Delta\alpha_j\right)\right]}{\sigma_i^2} + \sum_j \frac{\Delta\alpha_j^2}{\sigma_{\alpha_j}^2}$$

Caution: Most errors are provided as relative errors, a smaller value of cross section has smaller absolute error bias toward smaller averages Can be avoided by modified χ^2 definition: insert $\frac{M^{i,true}}{M^i}$

Discrepancy

- the PDF of gluon:

$$
\begin{aligned}
xg(x) &= Ax^B(1-x)^C(1+Dx) \\
&= [\mathbf{A}(\mathbf{x}^{\mathbf{B}} + \mathbf{Dx}^{\mathbf{1+B}})](1-X)^C
\end{aligned}
$$

- In the region of our data, these two forms are comparable